























Fig. 5. Angular dependence of the optimized asymmetric structures obtained in Fig. 4a.

#### 4. Conclusion

Lambertian models provide a simple and instructive intellectual framework for describing the effects of surface texturing on thin-film absorption. Even outside their range of validity (isotropic scattering), they can still be surprisingly descriptive: in our case, we find that the Lambertian prediction gives a rough guideline as to the optimal texture period, and is within 50% of the actual performance. In order to design an optimal texture structure at normal incidence, including strong anisotropic scattering, one must eventually depart from the confinements of analytical models, however beautiful, and resort to brute-force computation and parameter optimization. The results in this paper provide a glimpse of what is possible from such a computational approach, in which significant improvements are obtained even by a small number of local optimizations (despite the nonlinear and nonconvex nature of this optimization problem). Such local optima represent only lower bounds on the attainable performance, and by expending additional effort one could certainly envision pushing those bounds upwards, although locating the true global optimum with confidence seems daunting. It may also be possible to analytically prove more general upper bounds on performance for gratings with a given period and specific incident angle. As a practical matter, a more important goal is to adapt these techniques to 3D. In order to improve the efficiency of the calculation and make 3D optimization feasible, a number of techniques could be employed. First, one could use more sophisticated computational techniques than FDTD, such as boundary-element methods that only require the interfaces to be discretized [40]. Second, one could use adjoint methods to compute the gradient of  $F$  with respect to the optimization parameters [41], and thereby employ much more efficient gradient-based optimization methods (such methods have been used in topology optimization of photonic structures with hundreds or thousands of degrees of freedom [42–46]). As a heuristic method, it might be interesting to investigate using the optimized Fourier coefficients from the 2D simulations in this paper to form a two-dimensional texture with similar frequency components in 3D.

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