Slow-light enhancement of radiation pressure
in an omnidirectional-reflector waveguide

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We study the radiation pressure on the surface of a waveguide formed by omnidirectionally
reflecting mirrors. In the absence of losses, the pressure goes to infinity as the distance between the
mirrors is reduced to the cutoff separation of the waveguide mode. This divergence at constant
power input is due to the reduction of the modal group velocity to zero, which results in the
magnification of the electromagnetic field. Our structure suggests a promising alternative,
microscale system for observing the variety of classical and quantum-optical effects associated with
radiation pressure in Fabry–Perot cavities. © 2004 American Institute of Physics.

The group velocity of light can be dramatically slowed
in photonic crystals. Slow light enhances a variety of optical
phenomena, including nonlinear effects, phase-shift
sensitivity, and low-threshold laser action due to distributed
feedback gain enhancement. Here, we demonstrate that
slow light can also enhance radiation pressure. Using nu-
merical calculations, we show how a slow-light, photonic-
crystal structure incorporating omnidirectionally reflecting
mirrors can be used to maximize the radiation pressure be-
tween two surfaces.

For Fabry–Perot cavities, the effect of radiation pressure
on the motion of the mirror surfaces has been extensively
studied. The magnification of field intensity inside the cavity
results in strong optomechanical coupling, leading to inter-
esting classical effects such as optical bistability and quan-
tum optical effects such as the generation of nonclassical
states of light and the potential entanglement of macro-
scopic objects through optomechanical cooling. In the
Fabry–Perot system, the direction of light propagation is
fixed perpendicular to the mirror surfaces, and the reflectivity
of at least one mirror is necessarily less than one to allow
light to couple into the cavity. We introduce a generalized
system that lifts both restrictions and is particularly well
suited for embodiment in microscale devices.

The omnidirectional-reflector waveguide structure we
consider is formed by two omnidirectional mirrors separated
by a distance comparable to the wavelength. Light travelling
in the waveguide gives rise to a radiation pressure between
the mirror surfaces that depends on the intensity and the
mode characteristics. We first optimize the mirror structure to
maximize the radiation pressure for fixed electromagnetic
energy in the field. We then calculate the radiation pressure
as a function of distance as the mirrors are pushed together.
As the distance decreases, intensity buildup results from the
reduction of the group velocity of the waveguide mode. We
show that in the absence of losses, the force diverges at
waveguide cutoff for constant input power.

The system is shown in Fig. 1(a). Two semi-infinite
multilayer films are separated by an air region of thickness d.
Each of the films consists of a stack of alternating high- and
low-dielectric layers with period a. For definiteness, we take
the refractive indices to be n_h=3.45 and n_o=1.45, corre-
sponding to Si and SiO_2 at 1.55 µm. The relative layer thick-
nesses are determined by the quarter-wave condition
n_hi d_hi = n_o d_o, which we later show to be an optimal case, along
with the constraint that d_hi + d_o = a, yielding d_hi = 0.296a and
d_o = 0.704a. Waveguide modes can be supported in the air
region between the films. Superimposed on Fig. 1 is the funda-
mental guided mode, propagating in the direction shown.

It is sufficient to consider solutions of Maxwell’s equa-
tions that are uniform in the y direction. The electric field can
then be written in the form E(x, y, z, t) = Re[ E_{k,a}(z) e^{i(k_a x -\omega t)}],
where we have made use of the translational symmetry in the
x direction. The solutions can be divided into transverse elec-
tric (TE) modes (E||y and H⊥y), and transverse magnetic
(TM) modes (H||y and E⊥y). The TE dispersion relation is shown in Fig. 1(b), where the dimensionless quantity
\omega a/2 \pi c is plotted as a function of k a/2 \pi. Modes were com-
puted by planewave expansion. Shown in grey are the modes
of an infinite multilayer film system, or bulk modes. Colored
symbols indicate the dispersion relation for the funda-
mental waveguide mode for several values of d/a. A fre-
quency and wave vector within the band gap of the bulk
system indicates that the mode is guided in the air region
between the films. For each value of d/a, the group velocity
of the guided mode goes to zero at ka/2 \pi = 0. As d/a decreases,
the mode shifts down in frequency, and higher-order
modes will enter the band gap. We focus on the force on the
multilayer films produced by light travelling in the funda-
mental guided mode.

The force was calculated using a Maxwell stress-tensor
formalism. The time-averaged force is given by

\begin{align}
F_{\alpha} &= \int_S d\Omega \sum_\beta \frac{1}{8\pi} Re \left[ e^{i\delta_{\alpha\beta}} - \frac{1}{2} \partial_\beta \left[ \frac{1}{2} \partial_\alpha |H|^2 + \mu H^\dagger H \right] \right] n_\beta.
\end{align}

where the integral is taken over a surface S enclosing a vol-
ume V, which we take to be a box with parallel faces of area
A and z = 0. The outward normal to the surface is n. For
guided modes, the contribution from the face at Z goes to
zero as Z → ∞. The contribution from the sides of the box is...
classical arguments\(^{12}\) that, for a dielectric or metallic structure, modes can always be chosen such that either \(h_j\) or \(h_k\) is zero there. The time-averaged force per area \(A\) in the \(z\) direction on the upper film reduces to

\[
F/A = -\frac{1}{16\pi} \text{Re}(E_x^2E_x - E_y^2E_y - E_z^2E_z + H_x^2H_x - H_y^2H_y),
\]

(2)

To determine the optimal multilayer structure, we first consider the maximum achievable optical force for a fixed separation between the multilayer films and a fixed amount of energy in the electromagnetic field (as contrasted to fixed input power). The electromagnetic fields at a given frequency were calculated using planewave expansion.\(^2\) Figure 2(a) illustrates the dependence of the force on the frequency \(\omega\) and the period \(a\). We plot the dimensionless quantity \(Fd/U_{\text{field}}\) as a function of \(\omega a/2\pi\) for several values of \(a/d\), where \(F\) is the force due to area \(A\), and \(U_{\text{field}}\) is equal to the integral of the electromagnetic energy density over \(A\) and \(z\). The value of \(Fd/U_{\text{field}}\) is always positive: The effect of the light in the guided mode is to push the two films apart. For each value of \(a/d\), the force increases with decreasing frequency until it reaches the cutoff frequency of the mode, \(\omega(k=0)\) [see also Fig. 1(a)]. We note that the maximum value of the force \(F_{\text{max}}\) is independent of polarization, since TE and TM modes are degenerate at \(k=0\).

It can be shown by either quantum mechanical or classical arguments\(^{15}\) that, for a dielectric or metallic structure characterized by parameter shift \(\xi\), supporting a single mode at frequency \(\omega\) and wave vector \(k\), the time-averaged force on the dielectric is given by

\[
F = -\frac{1}{\omega} \left. \frac{\partial \omega}{\partial \xi} \right|_{k} U_{\text{field}},
\]

(3)

where it is assumed that the modal field decays to zero perpendicular to the propagation direction. It can be seen from Fig. 1(b) that for a given pair of multilayer films (fixed \(a\)), the frequency of the guided mode at a fixed wave vector changes most quickly with displacement \(d\) at \(k=0\) (i.e., the lines labeled by \(a/d\) are furthest apart at \(k=0\)), consistent with Fig. 2(a).

In Fig. 2(b), we plot \(F_{\text{max}}d/U_{\text{field}}\) as a function of \(\omega a/2\pi\), where \(F\) is the force due to area \(A\), \(U_{\text{field}}\) is the electromagnetic field energy contained in a slice of the waveguide that intersects the \(z\) axis in area \(A\), and \(\omega\) is the waveguide mode frequency. (b) Maximum force from (a) as a function of \(a/d\). The peak in the curve gives the maximum attainable force for fixed separation of the films \(d\) and energy in the fields \(U_{\text{field}}\), where the maximization is over the period of the films \(a\) and the frequency \(\omega\).

We next consider the distance dependence of the force. Suppose we choose the operating frequency \(\omega\) and the period \(a\) so as to maximize the force at distance \(d_o\) for fixed electromagnetic field energy \(U_{\text{field}}\). That is, we choose \(a = 0.45d_o\) and \(\omega = \omega(k=0, a/d_o = 0.45)\). As the separation is decreased from some initial value to \(d_o\), the force increases, as shown in Fig. 3(a). The group velocity of the waveguide mode at \(\omega\) simultaneously decreases to zero.

Now consider what happens at fixed input power, a more relevant constraint for any experimental realization of such a device. The energy in the field can be written as \(U_{\text{field}} = P_{\text{ext}}L/v_{\text{ext}}\), where \(L\) is the length in the propagation direction of the volume \(V\) with cross-sectional area \(A\). In Fig. 3(b), we plot the dimensionless quantity \(f(d/d_o) = F_{\text{max}}d/O_{\text{field}}\). As can be seen from the graph, \(f \rightarrow \infty\) as \(d \rightarrow d_o\). At fixed
frequency and input power, the force between the films is given by

\[ F(d) = f(d/d_o) \frac{P_m(L/c)}{d_o}, \]

and the force becomes infinite as the separation between the plates approaches \( d_o \). Physically, as the distance between the plates decreases, the group velocity of the waveguide mode decreases to zero. For a fixed power input, the light takes longer and longer to travel down the waveguide, and the magnitude of the field builds up, leading to a divergence in the force. For separations smaller than \( d_o \), the operating frequency is below the cutoff frequency of the mode, and the fields will decay evanescently along the waveguide, reducing the force.

In practice, the divergence at \( d_o \) will be removed by loss mechanisms, including losses due to finite mirror thickness, material absorption, input coupling, and scattering from disorder. An accurate numerical estimate of the achievable force will thus depend on a detailed analysis of these mechanisms.

Figure 3(c) shows one means of achieving a nearly constant power input for a range of separations. Here, a cylinder coated with a multilayer film forms the top surface of the waveguide. This geometry not only eases the alignment requirements between the two surfaces, but also forms an adiabatic input taper for coupling, e.g., a tightly focussed laser mode, to the guided mode of the waveguide. Due to the taper, a change in the minimum film separation will correspond to a much smaller fractional change in the film separation at the input, yielding a nearly constant input power. Moreover, any reflections from the output taper back into the waveguide will in fact increase the total force, as they increase the total amount of power in the waveguide.

For larger separations between the multilayer films, the central air waveguide will be multimode. The total force between the films is given by the sum over the force contributions due to the individual modes. As a result, the force as a function of distance will exhibit peaks at the cutoff distances of each of the higher-order modes. Unlike the case of radiation pressure in mirrored Fabry–Perot cavities, these peaks are assymmetric around the cutoff length.

Since the key condition for a force divergence is a zero-group-velocity mode at nonzero frequency, a divergence is also expected for metallic waveguides. In the absence of loss, the time-averaged force on the metallic waveguide walls is calculated to be

\[ \frac{F d}{P_m(L/c)} = \frac{\pi n c}{2 \omega} \frac{1}{\sqrt{d^2 - \left( \frac{\pi n c}{\omega} \right)^2}}, \]

for either TE or TM modes. (\( F = 0 \) for the transverse electromagnetic mode, since its frequency is independent of waveguide width.) The force at constant input power thus diverges as \( d \rightarrow \frac{\pi n c}{\omega} \) and the mode reaches cut off. However, to maximize the force, the distance between the plates (and consequently the operating wavelength) should be made as small as possible. Due to the high losses in metals at optical wavelengths, the force divergence should be far easier to observe in dielectric systems.

A number of applications of this work should follow from the ability to modify the mechanical oscillation of the structure by optical means, such as sensitive control and positioning on the microscale. A related effect may also occur for radiation pressure on particles. Operating our structure in reverse, by using an applied force to modify the waveguide group velocity, results in a tunable time-delay device. Moreover, since our omnidirectional-reflector waveguide may be viewed as a Fabry–Perot cavity operated via a sidewise-coupling scheme, it should provide an interesting alternate system in which to explore the rich collection of classical and quantum effects resulting from strong optomechanical coupling in Fabry–Perot systems.

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