Single-photon all-optical switching using waveguide-cavity quantum electrodynamics

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This paper demonstrates switching of a single signal photon by a single gating photon of a different frequency, via a cross-phase-modulation. This effect is mediated by materials exhibiting electromagnetically induced transparency (EIT), which are embedded in photonic crystals (PhCs). An analytical model based on waveguide-cavity QED is constructed for our system, which consists of a PhC waveguide and a PhC microcavity containing a four-level EIT atom. It is solved exactly and analyzed using experimentally accessible parameters. It is found that the strong coupling regime is required for lossless two-photon quantum entanglement.

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Several emerging technologies, such as integrated all-optical signal processing and all-optical quantum information processing, require strong and rapid interactions between two distinct optical signals [1]. Achieving this goal is a fundamental challenge because it requires a unique combination of large nonlinearities and low losses. The weak nonlinearities found in conventional media mean that large powers are required for switching. However, nonlinearities up to 12 orders of magnitude larger than those observed in common materials [2] with low losses can be achieved using materials exhibiting electromagnetically induced transparency (EIT) [2–4]. One can then envision inducing strong interactions between two very weak signals of different frequencies by placing a four-level EIT atom in a high-\(Q\) cavity, so that a very small signal at a specific atomic transition frequency could shift another resonant frequency of the system by a measurable amount [5]. This approach differs from several optical switching schemes for small numbers of photons that have previously been discussed in the literature. One of the pioneering papers in this area used a single three-level atom with a V-level structure in an optical cavity to induce a cross-phase-modulation of 16° between two photons [6]. EIT offers even further opportunities in terms of larger nonlinearities and greater tunability, which has directed much subsequent work in this direction. EIT materials have been predicted to cause a photon blockade effect, where the state of a cavity can be switched by the self-phase-modulation of a single photon [7–9] or several photons [10,11]. This effect has recently been observed experimentally [12]. Reference [13] predicts that ensembles of EIT atoms can be modulated to create quantum entangled states for a small number of photons. An alternative method is discussed in Ref. [14], whereby a laser beam can control the relative populations of a two-state system embedded in a photonic crystal (PhC), which switches its transmission properties at low power levels.

Reference [5] semiclassically demonstrates the strong interaction of very low intensity fields that can be mediated by EIT materials. This work extends that idea to the quantum regime by writing down the waveguide-cavity QED Hamiltonian for a system consisting of one or a few four-level EIT atoms strongly coupled to a PhC cavity mode, which in turn is coupled to a PhC waveguide, and solving it exactly. Furthermore, an approach to calculating the relevant parameters from first principles is demonstrated. It should be experimentally feasible, with EIT having already been demonstrated in a Pr-doped \(Y_2SiO_5\) crystal [15,16]. Note that compared to EIT systems, such as Na BECs displaying narrow bandwidths (e.g., 2 MHz [2]), switching can occur over much larger bandwidths even for single-photon power levels (e.g., 2 GHz, using the parameters from Ref. [17]) because the PhC cavity compensates for weaker nonlinearities, as demonstrated in this paper. Furthermore, this approach utilizes PhCs, which offer confinement of light to high-quality factor microcavities with low modal volumes, which facilitates strong coupling between light and matter. The emergence of phenomena associated with the quantization of the probe and gate fields (e.g., Rabi-splitting) is discussed. Finally, it is shown that switching behavior can be achieved with single probe and gate photons, and the physical parameters needed to achieve such operations are calculated.

Consider the following design, illustrated in Fig. 1. There is a cavity that supports two resonant modes, one with a resonant frequency \(\omega_{res}\) and the other with a control frequency \(\omega_{con}\), enclosing a single four-level EIT atom with coupling strengths \(g_{ij}\) and atomic transition frequencies \(\omega_{ij}\), where \(i\) and \(j\) refer to the initial and final atomic states, respectively. The EIT dark state is created by adding a classical coupling field to the cavity with frequency \(\omega_{23}\) and Rabi frequency \(2\Omega_c\); all other quantities are treated quantum me-

FIG. 1. (Color) Schematic illustration of the system investigated. A waveguide is coupled to a cavity with an EIT atom at its center. In the upper left-hand corner, an FDTD simulation that can be used to calculate the model parameters is shown.
channically. In general, any number of coupling schemes between the cavity and one or more waveguides could be utilized. However, in this paper, the $\omega_{\text{res}}$ cavity mode is side coupled to an adjacent single-mode waveguide with a radiative linewidth $\Gamma_w=\omega_{\text{res}}/2Q_w=V_w^2/v_w$, where $Q_w$ is the quality factor of the $\omega_{\text{res}}$ cavity mode, $V_w$ is the coupling strength, and $v_w$ is the group velocity in the waveguide—its dispersion relation $\omega(k)$ is assumed to be approximately linear near the $\omega_{\text{res}}$ resonance. For relatively strong cavity-waveguide couplings, radiative couplings out of the system are smaller and may be neglected. Also, the $\omega_{\text{res}}$ resonance is designed to have a much smaller decay rate $\Gamma_{\text{res}}=\omega_{\text{res}}/2Q_{\text{res}}$. This can be achieved by starting with two dipole modes, one with an even symmetry coupled strongly to the waveguide and one with an odd symmetry exactly decoupled from the waveguide. A slight shift in the cavity position can then create a slight coupling that, nonetheless, creates a substantial disparity in quality factors, i.e., $Q_{\text{res}} \gg Q_w$ (see, e.g., Refs. [18,19]). Alternatively, one could use two cavities to create even and odd modes with substantially different quality factors [20]. In the absence of an atom, this design produces a Lorentzian line shape for the reflection (because of the side coupling), centered around $\omega_{\text{res}}$ [21]. A PhC implementation of this is shown in the upper-left corner of Fig. 1—a triangular lattice of air holes in silicon with radius $0.48a$ that has a complete 2D photonic bandgap. A similar geometry has been used for quantum dots in PhC microcavities, as in Ref. [17]. That experimental setup exhibits a critical photon number $n_c=1L^2/\pi g^2=0.55$ and critical atom number $N_0=2\Gamma_L^2G^2=4.2$. Ideally, both of these numbers would be $<1$ for quantum information processing [22]. It should be possible to achieve this goal with improvements in $Q$ or modal volume $V_{\text{mode}}$, or by placing several atomic or quantum dot systems in the same microcavity. Note that it could also be possible to achieve similar behavior with other physical systems, such as high-finesse Fabry-Perot optical microcavities [23], or ultrahigh-$Q$ toroidal microresonators [24].

Combining Ref. [8]'s Hamiltonian for an EIT atom in a cavity and Ref. [25]’s Hamiltonian for a waveguide interacting with a cavity yields:

$$H/\hbar = \sum_k \omega_k a_k^\dagger a_k + \omega_{\text{res}} a^\dagger a + \omega_{\text{con}} b^\dagger b + \sum_k V_w (a_k^\dagger + a_k) (a^\dagger + a) + \omega_{13} \sigma_{22} + (\omega_{14} - i\Gamma_{\text{res}}) \sigma_{33} + (\omega_{14} - i\Gamma_{\text{con}}) \sigma_{44} + \Omega \sigma_{32} \cos(\omega_{\text{res}} t) + g_{13} (a^\dagger \sigma_{13} + a \sigma_{31}) + g_{23} (b^\dagger \sigma_{23} + b \sigma_{32})$$

where $a_k$ are the annihilation operators for waveguide states of wave vector $k$ and frequency $\omega_k$; $a$ and $b$ are the annihilation operators for cavity photon states of frequencies $\omega_{\text{res}}$ and $\omega_{\text{con}}$ respectively (which are considered in this paper to be unoccupied or singly occupied); $\sigma_{ij}$ are the projection operators that take the atomic state from $j$ to $i$; $\Gamma_{\text{res}}$ is the nonradiative decay rate of the third level; $\Gamma_{\text{con}}$ is the nonradiative decay rate of the fourth level; $\Delta_{\text{res}}=\omega_{\text{res}}-\omega_{\text{con}}-i\Gamma_{\text{res}}$ is the complex detuning of the $1\rightarrow 3$ transition from $\omega_{\text{res}}$; and $\Delta_{\text{con}}=\omega_{\text{con}}-i\Gamma_{\text{con}}$ is the complex detuning of the $2\rightarrow 4$ transition from $\omega_{\text{con}}$. In this paper, the cavity resonance is designed to match the $1\rightarrow 3$ transition, i.e., $\omega_{\text{res}}=\omega_{13}$, so that $\Delta_{\text{res}}=i\Gamma_{\text{res}}$. Also, although $\Delta_{\text{res}}$ is predominantly real, in general, there is an imaginary part corresponding to absorption losses in the fourth level. However, when the detuning greatly exceeds the decay rate of the upper level, this contribution may be neglected. Losses from the second atomic level are also neglected, since, typically, it is a metastable state close to the first atomic level in energy. Finally, although, in general, the two cavity modes will have at least slightly different frequencies, we set $\omega_{\text{con}}=\omega_{\text{res}}$ for simplicity.

The Hamiltonian in Eq. (1) can then be rewritten in real space and separated into a diagonal part

$$H/\hbar = \omega_{\text{res}} \int dx [a_R^\dagger(x) a_R(x) + a_L^\dagger(x) a_L(x)] + \omega_{\text{res}} (a^\dagger a + b^\dagger b + \sigma_{33} + \sigma_{44}) + \omega_{21} (\sigma_{22} + \sigma_{44}),$$

where $a_R$ and $a_L$ refer to left and right moving waveguide photons, respectively, as well as an interaction part

$$H/\hbar = \int dx [a_R^\dagger(x) (-i v_g \sigma_{31} - \omega_{\text{res}} a_R^\dagger(x) + a_L^\dagger(x) (iv_g \sigma_{31})$$

$$\omega_{\text{res}} a_L(x) + V_w \delta(x) [a_R^\dagger(x) a + a_R(x) a^\dagger + a_L^\dagger(x) a^\dagger + a_L(x) a^\dagger] + \Omega \sigma_{32} + g_{13} (a^\dagger \sigma_{13} + a \sigma_{31})$$

$$i\Gamma_{\text{res}} \sigma_{33} + \Delta \sigma_{44} + g_{23} (b \sigma_{42} + b^\dagger \sigma_{23})$$

via the interaction picture (using the rotating-wave approximation [26]), where the total system Hamiltonian is given by $H=H_0+H_I$. The eigenstate for the system can be written as

$$|\psi_k\rangle = \left\{ e^{i \delta_k x} \right\}_{\text{phc}} \otimes |k\rangle_{\text{atom}}$$

where

$$\delta_k(x) = e^{i k x} [\theta(-x) + i \theta(x)] \right\}_{\text{phc}} \otimes |1\rangle_{\text{atom}}$$

$e_k$ is the probability amplitude of the cavity photon at $\omega_{\text{res}}$, and $f_k$, $h_k$, and $p_k$ are the occupations of the third, second, and fourth atomic levels, respectively. $t$ and $r$ are the waveguide transmission and reflection amplitudes, respectively. All of these parameters are determined when the eigen equation is solved below. $|0,0,1\rangle_{\text{phc}} \otimes |1\rangle_{\text{atom}}$ is an eigenstate consisting of a direct product of a photonic state (phc) and an atomic state (atom). The photonic state consists of zero photons in the waveguide, zero photons in the cavity at $\omega_{\text{res}}$, and one photon in the cavity at $\omega_{\text{con}}$, respectively. The atomic state consists of a single atom in its ground state. Note that $|\psi_k\rangle$ is written in terms of an annihilation operator $b$ in order to simplify the notation, which would otherwise require $b^\dagger$ operators in all but one term.

Applying the Hamiltonian [Eq. (3)] to the time-independent eigenvalue equation $H/\hbar |\psi_k\rangle = \hbar \epsilon_k |\psi_k\rangle$, where
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FIG. 2. (Color) Waveguide reflection for a lossless three-level EIT atom for the four labeled values of the atomic coupling strength $g_{13}$ (in gigahertz). The radiation rate $\Gamma_{w}=21.5$ GHz and the ratio $g_{13}/\Omega_{c}=2$ are fixed. Larger $g_{13}$ produces larger peak separations (the blue curve shows Rabi peaks outside of the plot), favorable for switching.

$$\epsilon_{e}=\omega-\omega_{\text{res}}$$

and solving for the reflection coefficient yields 

$$\left|\mathcal{R}(\epsilon_{e})\right|^{2} = \frac{\Gamma_{w}}{\Omega_{c}} \left(\frac{\epsilon_{e}-\Gamma'_{3}}{\epsilon_{e}-\epsilon_{2}^{\pm}(\epsilon_{e}-\Delta\omega_{a})}\right)^{2}$$

where 

$$\xi = \epsilon_{e} - \frac{g_{13}^{2}}{\epsilon_{e} + i\Gamma'_{3} - \frac{\epsilon_{e}^{2}}{\epsilon_{2}^{\pm}(\epsilon_{e}-\Delta\omega_{a})}}$$ (6)

The parameters $g_{13}$, $V_{w}$ (or $\Gamma_{w}$), $\epsilon_{2}^{\pm}$, and $\Omega_{c}$ of Eq. (3) can be determined from a numerical solution to Maxwell’s equations (as in Ref. [27]) as follows. First, the cavity mode is excited by a source, and the modal volume of the cavity is found from the field patterns by $V_{\text{mode}}=(\int_{\text{mod}} |\mathcal{E}|^{2} d\mathbf{x})/\mathcal{E}_{\text{max}}^{2}$. One can then apply the formula $g_{13}=\sqrt{\pi e^{2}f_{13}^{2}m\epsilon V_{\text{mode}}}$ [28], where $\epsilon$ is the elementary electric charge, $e$ is the dielectric constant of the medium in which the atomic system is embedded, $m$ is the free electron mass, and $f_{13}$ is the oscillator strength for the $|1\rangle \rightarrow |3\rangle$ transition (1/2 in Na[2]). The linewidth $\Gamma_{w}$ can be calculated by examining the decay rate of the field in the cavity mode. The waveguide group velocity is given by $v_{g}=[d(\omega/k)]_{\omega=\omega_{\text{res}}}$.

Finally, the Rabi frequency $\Omega_{c}$ can be estimated from quantum mechanics by first determining the vacuum Rabi splitting for the $2 \rightarrow 3$ atomic transition $g_{23}$, and then multiplying by $\sqrt{n}$, where $n$ is the number of $\omega_{13}$ photons.

First, consider the case of a two-level atomic system (i.e., $\Omega_{c}=0$, $g_{24}=0$), with a waveguide coupling $\Gamma_{w}$ and a nonradiative decay rate $\Gamma_{3}$. For a fixed atom-photon coupling $g_{13}$ and zero nonradiative absorption, the single resonant mode at $\epsilon_{e}=0$ experiences a Rabi splitting into two orthogonal linear superpositions of the cavity and atom modes at $\epsilon_{e}=\pm g_{13}$. As long as one remains in the strong coupling regime $g_{13}>\Gamma_{3}-\Gamma_{w}$, the absorption for all frequencies increases nearly linearly with $\Gamma_{3}$ for small $\Gamma_{3}$ [29].

However, in the opposite regime of weak coupling ($g_{13}<\Gamma_{3}-\Gamma_{w}$), the normal modes of the system are mostly photonic (lossless) or mostly atomic (very lossy). This phenomenon eliminates the Rabi splitting and gives rise to a reflection nearly indistinguishable from a system without an atom for sufficiently large $\Gamma_{3}$.

Now, consider a three-level atomic system without losses in the strong coupling regime. Compared to the two-level system, a third mode, corresponding to the dark state of the EIT atom, will emerge at $\epsilon_{e}=0$ between the previously observed Rabi-split peaks. The dark eigenstate is given by $|\phi_{\text{dark}}\rangle=[a^{\dagger}-g_{13}/\Omega_{c}]|0,0,0\rangle_{\text{phc}} \otimes |\text{atom}\rangle$. The width of the central peak is expected to scale as $(\Omega_{c}/g_{13})^{2}$ for small $\Omega_{c}/g_{13}$ [13]. If one substitutes the expression given in Ref. [28] for $g_{13}$, one obtains the classical results found in Refs. [2,5,30]. Meanwhile, the width of the side peaks is set by $\Gamma_{w}$ and remains roughly constant as one tunes the parameters of the system.

In Fig. 2, $g_{13}/\Omega_{c}=2$ while $g_{13}$ is varied. It is shown that as $g_{13}$ is decreased, the central resonance width stays constant, while the distance between the central and Rabi-split peaks becomes smaller. For use in applications, it therefore seems optimal to have a large Rabi splitting, corresponding to the very strong coupling limit, which can also be viewed as corresponding to critical photon and atom numbers much less than 1. The experimental values for a system with a single quantum dot emitting a single photon observed in Ref. [17] correspond to a regime where $g_{13}=\Gamma_{w}$ — specifically, they find that for operation at $\lambda=1.182$ $\mu$m, $g_{13}=20.5$ GHz and $\Gamma_{w}=21.5$ GHz; note that PhC microcavities are optimal for simultaneously decreasing $\Gamma_{w}$ and increasing $g_{13}$.

Now, consider a four-level system with a control photon present. Two possible effects can be induced by the control photon. When the control frequency $\omega_{\text{con}}$ is close to the electronic transition frequency $\omega_{24}$, an Autler-Townes doublet is observed; upon detuning, an AC-Stark shift will be induced in this system instead [5,8]. The latter effect has been suggested as a switching mechanism in Refs. [5,30,31]. This can

FIG. 3. (Color) Waveguide reflection (blue) and absorption (red) in the absence (solid) and presence (dashed) of an control photon, demonstrating nonlinear single-photon switching ($\Gamma'_{w}=21.5$ GHz, $g_{13}=20.5$ GHz, $\Omega_{c}=2$ GHz, $\Gamma_{3}=30$ GHz, $g_{24}=8$ GHz, and $\Delta\omega_{24}=30$ GHz).

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First, in the regime where the presence or absence of one control photon. In order to make this scheme viable, one can take two different approaches. First, in the regime where \( g_{13} \approx \Gamma_w \), it is possible to switch the peak reflection frequency by an amount greater than the EIT-narrowed central peak width. A second, less obvious approach, appropriate if producing a large nonradiative decay \( \Gamma_3 \) or small \( \Omega_r \), is difficult in a single-atom device, is to enhance the ratio \( g_{13} / \Gamma_w \). This goal can be achieved by either decreasing \( \Gamma_w \) or \( V_{mode} \), or by increasing the number of atoms from one to \( N \). The first example of switching by decreasing the waveguide coupling is shown in Fig. 4, where the waveguide coupling width \( \Gamma_w \) is decreased by about a factor of 7 to \( \Gamma_w = 3 \) GHz. Now the peaks are narrow enough that a single photon of frequency \( \omega_{\text{con}} \) can shift the peak by more than the full width at half maximum. The second example of switching, by increasing the number of atoms is illustrated in Fig. 5. In general, it is clear that increasing the number of atoms will improve the coupling strength; the \( N \)-atom treatment in Ref. [32] shows that the coupling constant \( g_{13} \rightarrow g'_{13} = g_{13} \sqrt{N} \). Furthermore, one can generalize the arguments of Ref. [32] to a four-level system of \( N \) atoms to show that the other coupling constants \( g_{24} \) and \( \Omega_r \) will scale in an identical fashion (i.e., \( g_{24} \rightarrow g'_{24} = g_{24} \sqrt{N} \), \( \Omega_r \rightarrow \Omega'_r = \Omega_r \sqrt{N} \)). This collective Rabi oscillation separates the Rabi-split peaks much further from the central peak. Figure 5 shows switching exploiting this phenomenon based on parameters from Ref. [17] and using \( N=49 \). The advantage of this lossless switching scheme is that one obtains a substantially greater tuning range and contrast (the difference between the peaks and the troughs) than with the lossy (\( \Gamma_3 \neq 0 \)) scheme.

In conclusion, the reflection peak of a waveguide-cavity system can be switched in and out of resonance by a single gating photon, assuming realistic experimental parameters. Thus, one photon can be used to gate another photon of a different frequency, via a Kerr cross-phase-modulation. This approach is distinct from the photon blockade system where self-phase-modulation is responsible for the switching behavior. Under proper circumstances, this can give rise to two-photon entangled states. The integration of microcavities and waveguides in the same photonic crystal means that the entanglement could be preserved, in principle, throughout the system, which could be of use for quantum information processing [22].

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