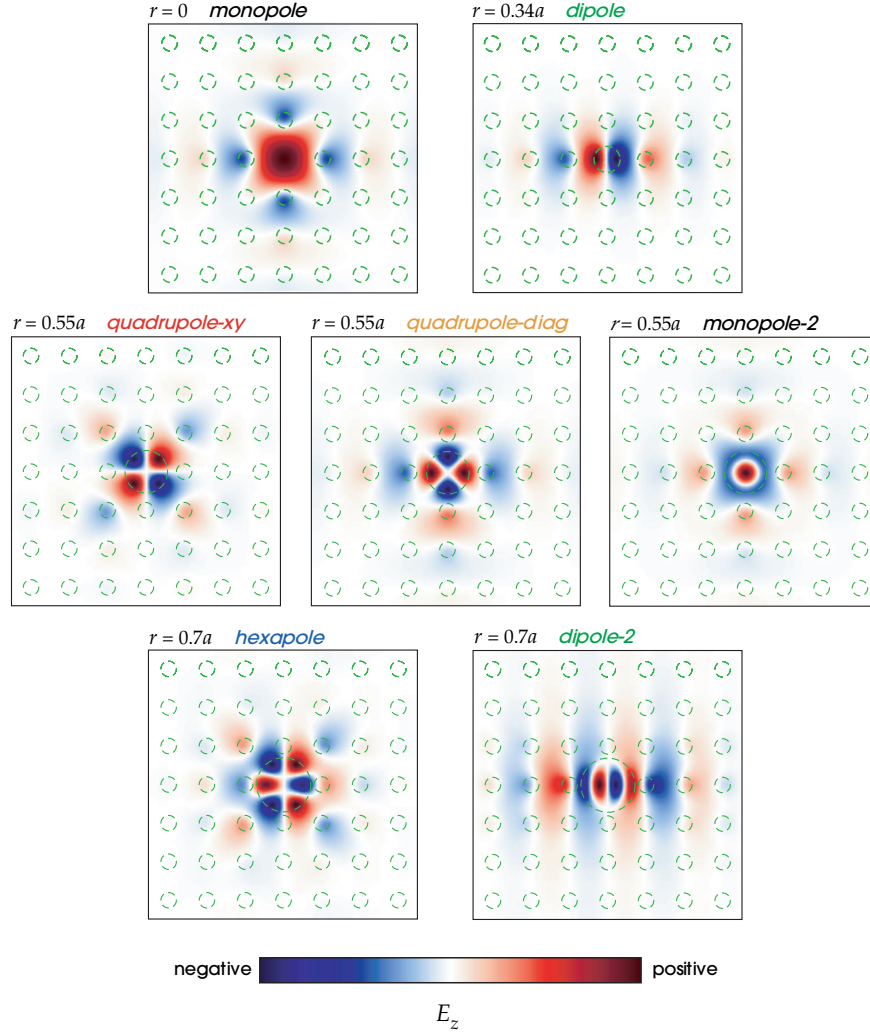**Figure 1:** The photonic band structure for a square array of dielectric columns with $r = 0.2a$. The blue bands represent TM modes and the red bands represent TE modes. The left inset shows the Brillouin zone, with the irreducible zone shaded light blue. The right inset shows a cross-sectional view of the dielectric function. The columns ($\varepsilon = 8.9$, as for alumina) are embedded in air ($\varepsilon = 1$).
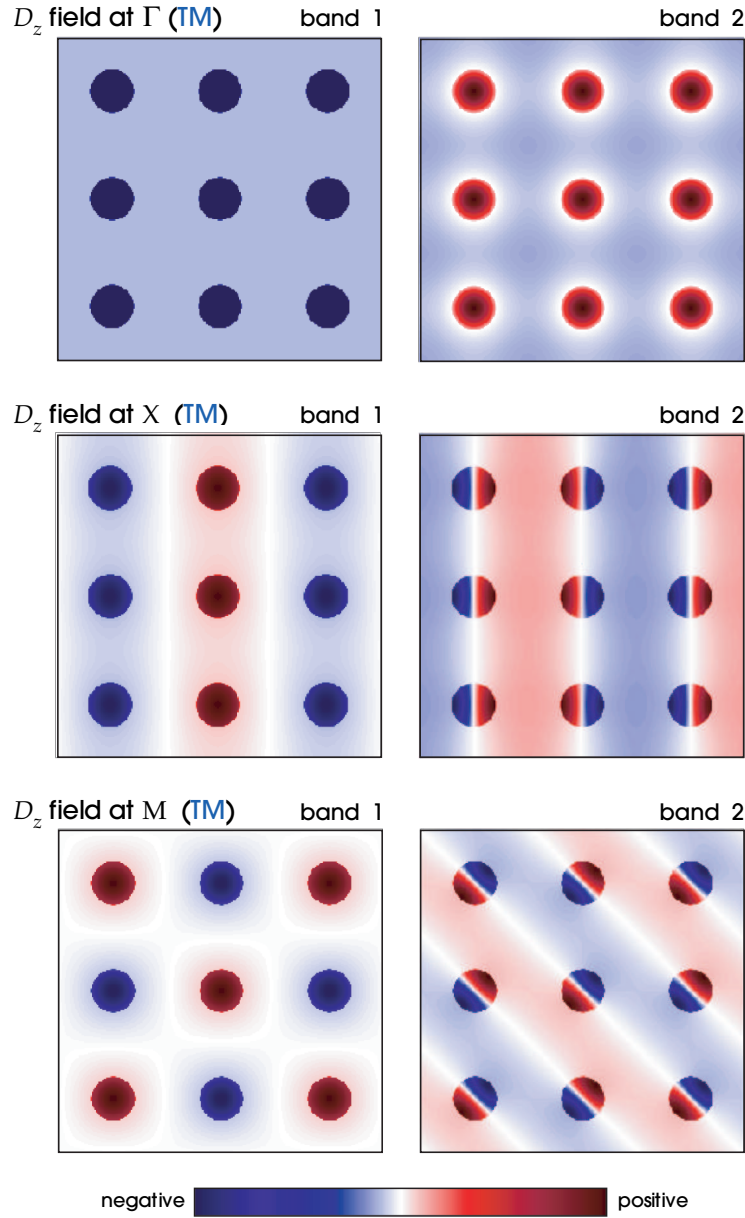
Figure 2: *Displacement* fields of TM states inside a square array of dielectric ($\varepsilon = 8.9$) columns in air. The color indicates the amplitude of the displacement field, which points in the z direction. Modes are shown at the Γ point (top), the X point (middle), and the M point (bottom). In each set, the dielectric band is on the left and the air band is on the right. The fields of the air band at the M point are from one of a pair of degenerate states.
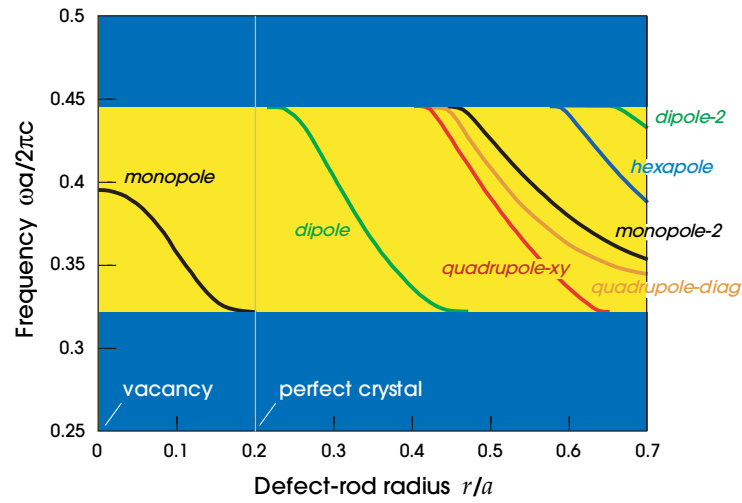
Figure 3: The evolution of localized modes associated with a defect column in an otherwise perfect square lattice with the parameters of figure 1, as the defect's *radius* is changed. A radius of $r = 0.2a$ corresponds to the perfect crystal. The TM band-gap frequency range, about 0.32–0.44 $2\pi c/a$, is shaded yellow. If the radius is *decreased* then a single *monopole* state is pushed *up* into the gap. If the radius is *increased* then a sequence of higher-order modes (with more nodal planes) are pulled *down* into the gap. The *dipole*, *hexapole*, and *dipole-2* states are doubly degenerate, while the other states are nondegenerate. The field patterns for these states and explanations for their names are in figure 4.

$r = 0$ *monopole*

$r = 0.34a$ *dipole*

$r = 0.55a$ *quadrupole-xy*

$r = 0.55a$ *quadrupole-diag*

$r = 0.55a$ *monopole-2*

$r = 0.7a$ *hexapole*

$r = 0.7a$ *dipole-2*

negative   positive

E_z

Figure 4: Electric-field (E_z) patterns of states localized about a defect in a square lattice of rods in air, where the radius r of a single rod has been changed. The rods are shown as dashed green outlines. *Top left:* a **monopole** state formed by completely removing a rod ($r = 0$), which has a single lobe in the defect and high symmetry. *Top right:* **dipole** state for an increased radius $r = 0.34a$, which is doubly degenerate with the other state (not shown) formed by a 90° rotation. *Middle row:* three nondegenerate states formed by an increased radius $r = 0.55a$. There are two **quadrupole** states, with the left (*quadrupole-xy*) having nodal planes along the x and y axes and transforming like $f(\boldsymbol{\rho}) = xy$ under rotations, while the center (*quadrupole-diag*) state has diagonal nodal planes and transforms like $x^2 - y^2$. The *monopole-2* state at right is a second-order monopole state, with an extra nodal plane in the radial direction. *Bottom row:* two states formed by an increased radius $r = 0.7a$, *each* of which is doubly degenerate with another state (not shown) formed by a 90° rotation. The **hexapole** state has six lobes in the defect, while *dipole-2* is a second-order dipole pattern.

4

E_z
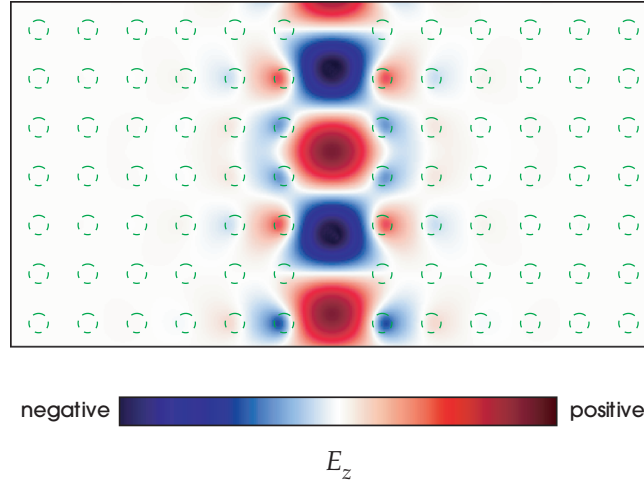
Figure 5: Electric-field (E_z) pattern associated with a linear defect formed by removing a column of rods from an otherwise-perfect square lattice of rods in air. The resulting field, shown here for a wave vector $k_y = 0.3\,(2\pi/a)$ along the defect, is a **waveguide mode** propagating along the defect. The rods are shown as dashed green outlines.
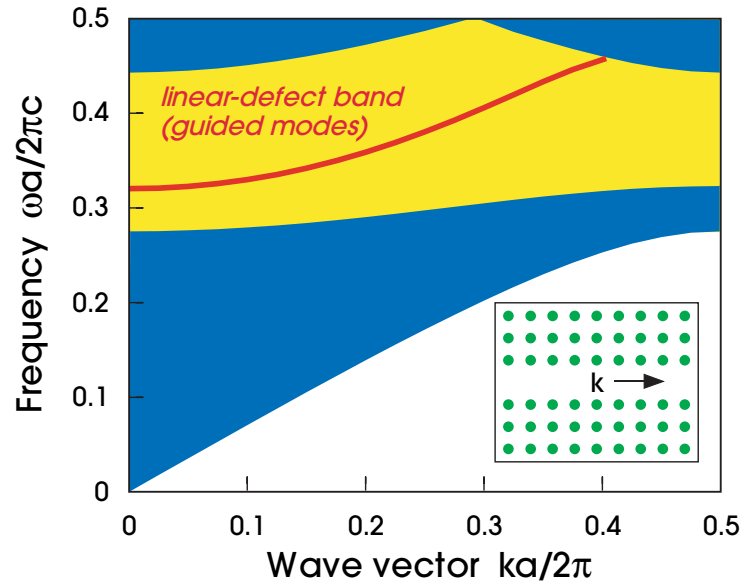
Figure 6: The projected band structure of the line defect (inset) formed by removing a row (or column) of rods from an otherwise perfect square lattice from figure 1, plotted versus the wave vector component k along the defect. The extended modes in the crystal become continuum regions (blue), whereas inside the band gap (yellow) a defect band (red) is introduced corresponding to a localized state as in figure 5.
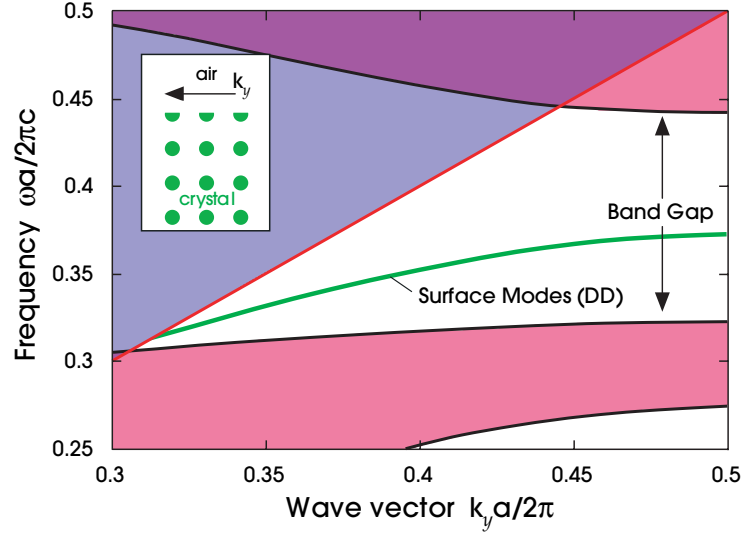
Figure 7: The projected band structure of the constant-x surface of the square lattice of alumina rods in air, with the termination shown in the inset. The line in the gap corresponds to a surface band in which light is exponentially localized to the surface (green, DD).
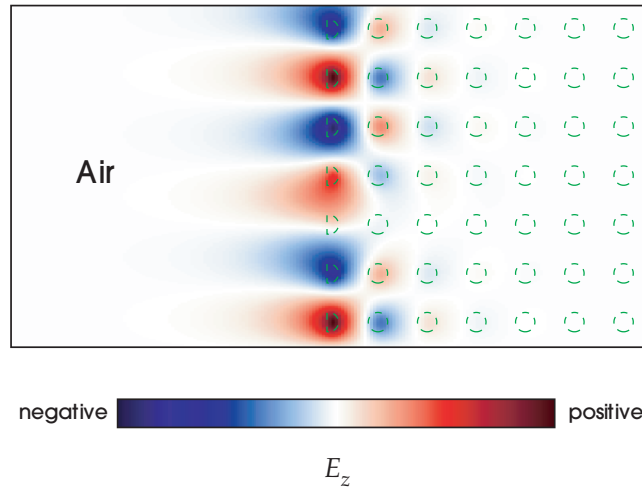
Figure 8: Electric-field (E_z) pattern associated with a surface-localized state formed by truncating a square lattice of alumina rods in air, cutting each rod in half at the boundary as in figure 7. The mode shown corresponds to a surface-parallel wave vector $k_y = 0.4\,(2\pi/a)$. The dielectric rods are shown as dashed green outlines.