18.369 Problem Set 5

Due Monday, 9 April 2007.

Problem 1: Dispersion

Derive the width of a narrow-bandwidth Gaussian pulse propagating in 1d (x) in a dispersive medium, as a function of time, in terms of the dispersion parameter $D = \frac{2\pi c}{v_g^2 \lambda^2} \frac{dv_g}{d\omega} = -\frac{2\pi c}{\lambda^2} \frac{d^2 k}{d\omega^2}$ as defined in class. That is, assume that we have a pulse whose fields can be written in terms of a Fourier transform of a Gaussian distribution:

$$\text{fields} \sim \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-(k-k_0)^2/2\sigma^2} e^{i(kx-\omega t)} dk,$$

with some width σ and central wavevector $k_0 \gg \sigma$. Expand ω to second-order in k around k_0 and compute the inverse Fourier transform to get the spatial distribution of the fields, and define the "width" of the pulse in space as the standard deviation of the $|\text{fields}|^2$. That is, width $= \sqrt{\int (x-x_0)^2 |\text{fields}|^2 dx / \int |\text{fields}|^2 dx}$, where x_0 is the center of the pulse (i.e. $x_0 = \int x |\text{fields}|^2 dx / \int |\text{fields}|^2 dx$).

You should be able to show, as argued in class, that D asymptotically (after a long time, or equivalently for large x_0) gives the pulse spreading in time per unit distance per unit wavelength (bandwidth).

Problem 2: Fabry-Perot Waveguides

(a) Starting with the bandgap1d.ctl MPB control file from problem set 4, which computes the frequencies as a function of k_x. Modify it to compute the frequencies as a function of k_y for some range of k_y (e.g. 0 to 2, in units of $2\pi/a$ ... recall that the k_y Brillouin zone is infinite!) for some fixed value of k_x, and to use $\varepsilon_2 = 2.25$ instead of 1.[1]

Compute and plot the TM projected band diagram for the quarter-wave stack with ε of 12 and 2.25. That is, plot $\omega(k_y)$ for several bands, first with $k_x = 0$, then $k_x = 0.1$, then 0.2, then ... then 0.5, and interpolate intermediate k_x to shade in the "continuum" regions of the projected bands. Verify that the extrema of these continua lie at either $k_x = 0$ or $k_x = 0.5$ (in units of $2\pi/a$), i.e. at the B.Z. edges.

(b) Modify the MPB defect1d.ctl file from problem set 4 to compute the defect mode as a function of k_y (for $k_x = 0$).

Changing a single ε_2 layer by $\Delta\varepsilon = 4$, with an $N = 20$ supercell, plot the first 80 bands as a function of k_y for some reasonable range of k_y. Overlay your TM projected band diagram from part (a), above, to show that the bands fall into two categories: modes that fall within the projected "continuum" regions from part (a), and discrete guided bands that lie within the empty spaces. (If there are any bands *just* outside the *edge* of the continuum region, increase the supercell size to check whether those bands are an artifact of the finite size.) Plot the fields for the guided bands (a couple of nonzero k_y points will do) to show that they are indeed localized.

(c) Modify the defect structure and plot the new band diagram(s) (if necessary) to give examples of:

(i) "index-guided" bands that do not lie within a band gap

(ii) a band in the gap that intersects a continuum region

(iii) a band in the gap that asymptotically approaches, but does not intersect, the continuum regions

[1] You might want to add a "kx" parameter via "(define-param kx 0)" so that you can change k_x from the command line with "mpb kx=0.3 ...".