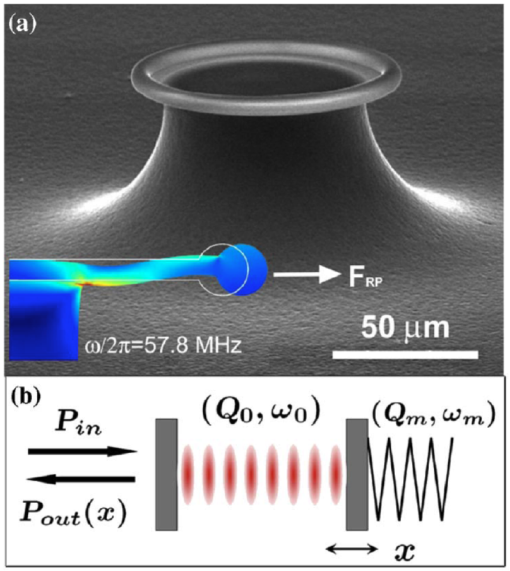
Understanding Resonant Systems



[Schliesser et al., *PRL* **97**, 243905 (2006)]

- Option 1: Simulate the whole thing exactly
  - — many powerful numerical tools
  - — limited insight into a single system
  - — can be difficult, especially for weak effects (nonlinearities, etc.)

- Option 2: Solve each component separately, couple with explicit perturbative method (one kind of "coupled-mode" theory)

- Option 3: abstract the geometry into its most generic form
  …write down the *most general* possible equations
  …constrain by fundamental laws (conservation of energy)
  …solve for universal behaviors of a whole class of devices
  … characterized via specific parameters from option 2

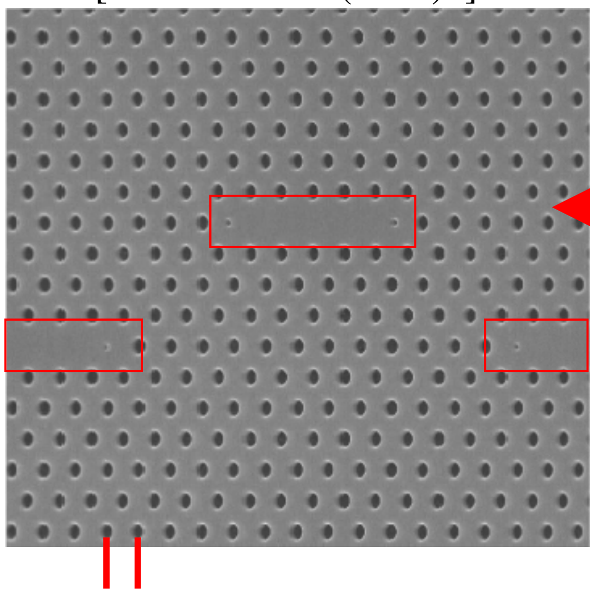
"Temporal coupled-mode theory"

- Generic form developed by Haus, Louisell, & others in 1960s & early 1970s, many variations…
  - Haus, *Waves & Fields in Optoelectronics* (1984)
  - Very general description/derivation: Suh, Wang, & Fan (2004)
  - Reviewed in our *Photonic Crystals: Molding the Flow of Light*, 2nd ed., ab-initio.mit.edu/book
- Equations are generic ⇒ reappear in many forms in many systems, rederived in many ways (e.g. Breit–Wigner scattering theory)
  - full generality is not always apparent

(modern name coined by S. Fan @ Stanford)

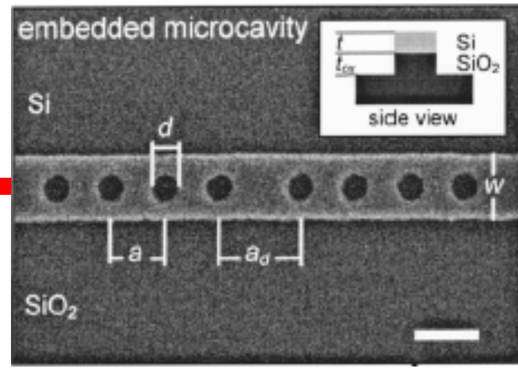
TCMT example: a linear filter

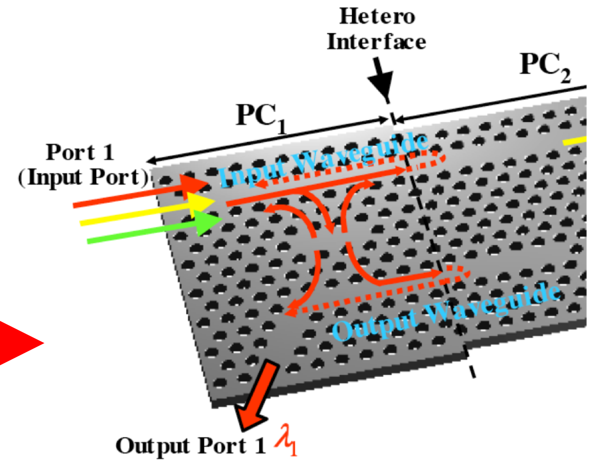
[Notomi *et al*. (2005).]



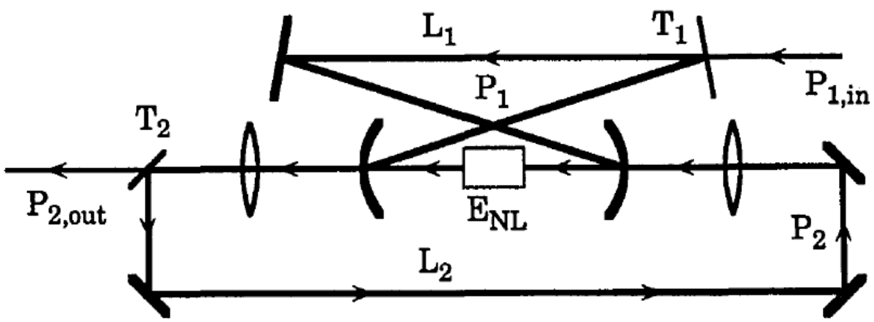
420 nm

[C.-W. Wong, *APL* **84**, 1242 (2004).]





[Takano *et al*. (2006)]

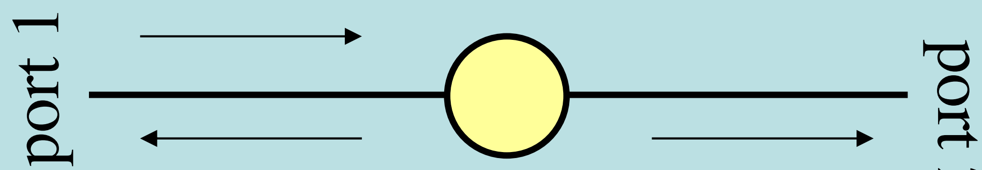


[Ou & Kimble (1993)]

= *abstractly:*

two single-mode i/o ports

+ one resonance

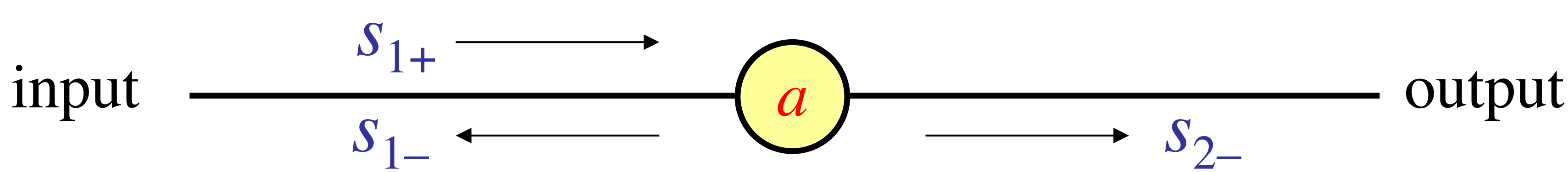


resonant cavity

frequency ω_0, lifetime τ

Temporal Coupled-Mode Theory

for a linear filter



resonant cavity
frequency ω_0, lifetime τ

$|s|^2$ = power

$|a|^2$ = energy

$$\frac{da}{dt} = -i\omega_0 a - \frac{2}{\tau} a + \sqrt{\frac{2}{\tau}} s_{1+}$$

$$s_{1-} = -s_{1+} + \sqrt{\frac{2}{\tau}} a, \quad s_{2-} = \sqrt{\frac{2}{\tau}} a$$

assumes only:

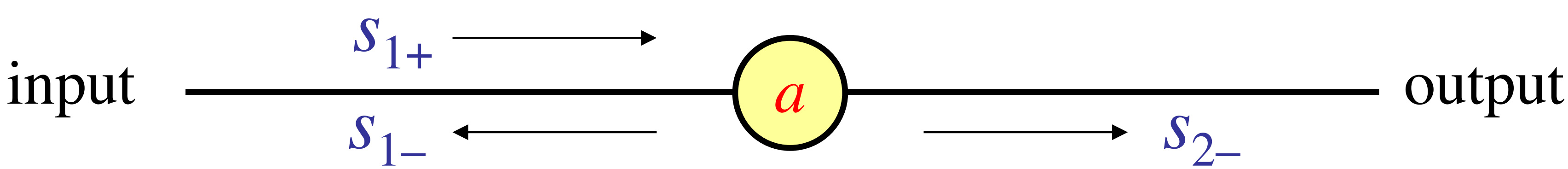
- exponential decay (strong confinement)

$\left.\begin{array}{r} \\ \textit{can be} \\ \textit{relaxed} \\ \\ \end{array}\right\{$

- linearity
- conservation of energy
- time-reversal symmetry

Temporal Coupled-Mode Theory

for a linear filter

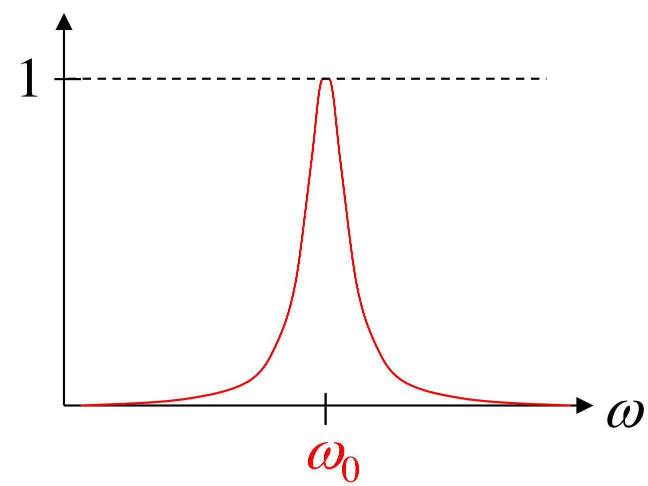


resonant cavity
frequency ω_0, lifetime τ

$|s|^2$ = flux

$|a|^2$ = energy

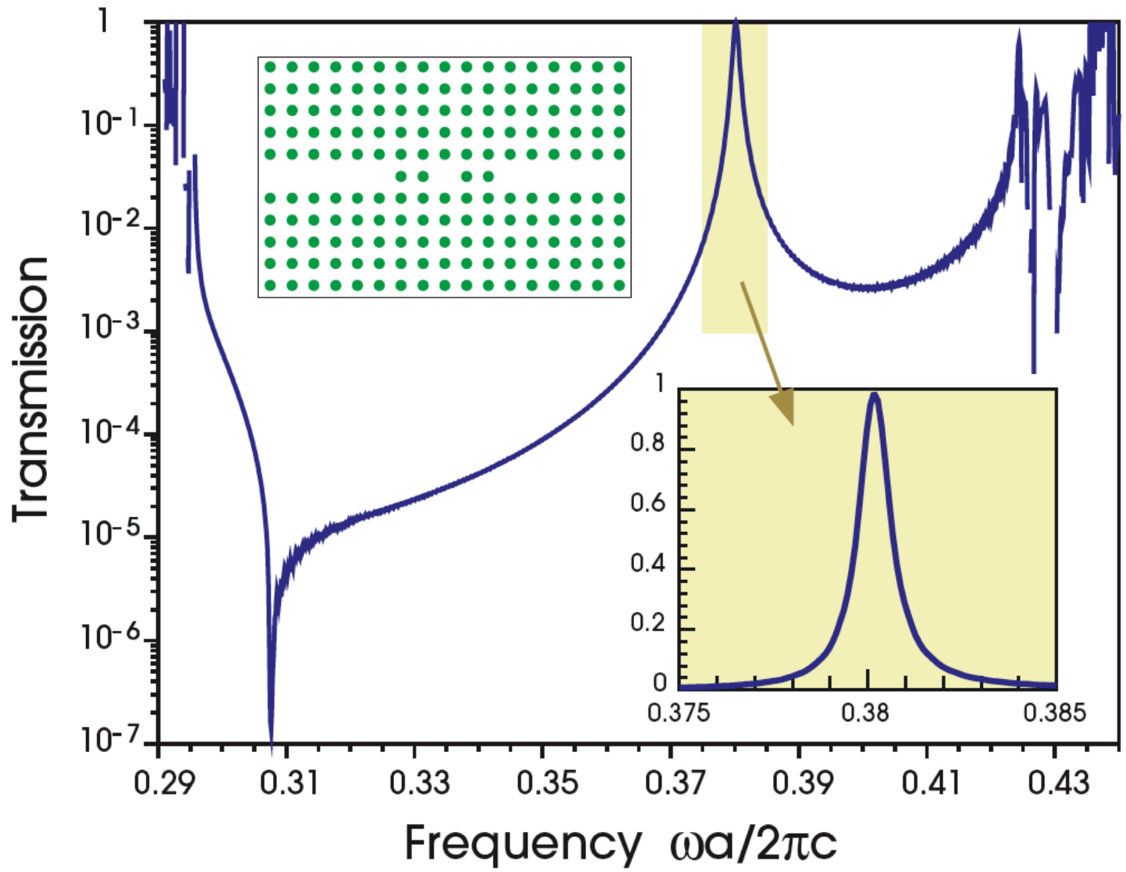
transmission T
$= | s_{2-} |^2 / | s_{1+} |^2$



T = Lorentzian filter

$$= \frac{\frac{4}{\tau^2}}{(\omega - \omega_0)^2 + \frac{4}{\tau^2}}$$

Resonant Filter Example

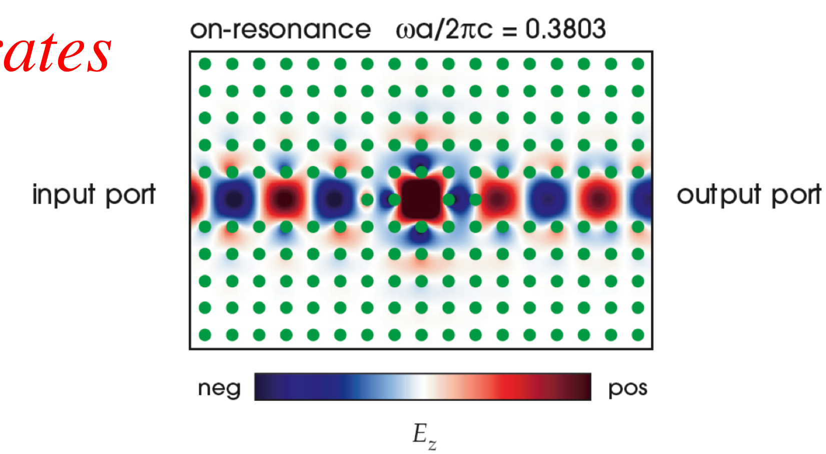


Lorentzian peak, as predicted.

An apparent miracle:

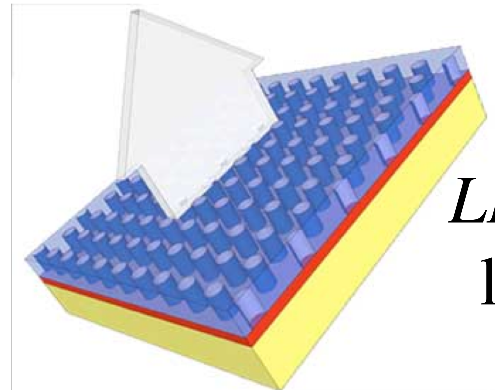
~ 100% transmission
at the resonant frequency

cavity decays to input/output with *equal rates*
– At resonance, reflected wave
destructively interferes
with backwards-decay from cavity
& the two *exactly cancel*.

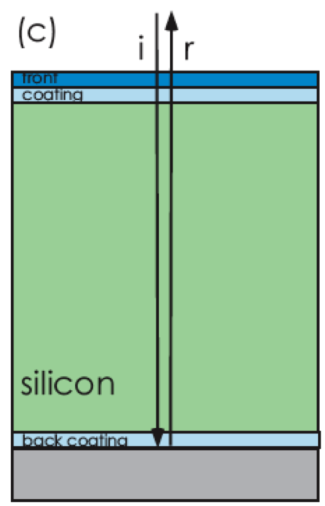


Some interesting resonant transmission processes

Wireless resonant power transfer
[M. Soljacic, MIT (2007)]
witricity.com



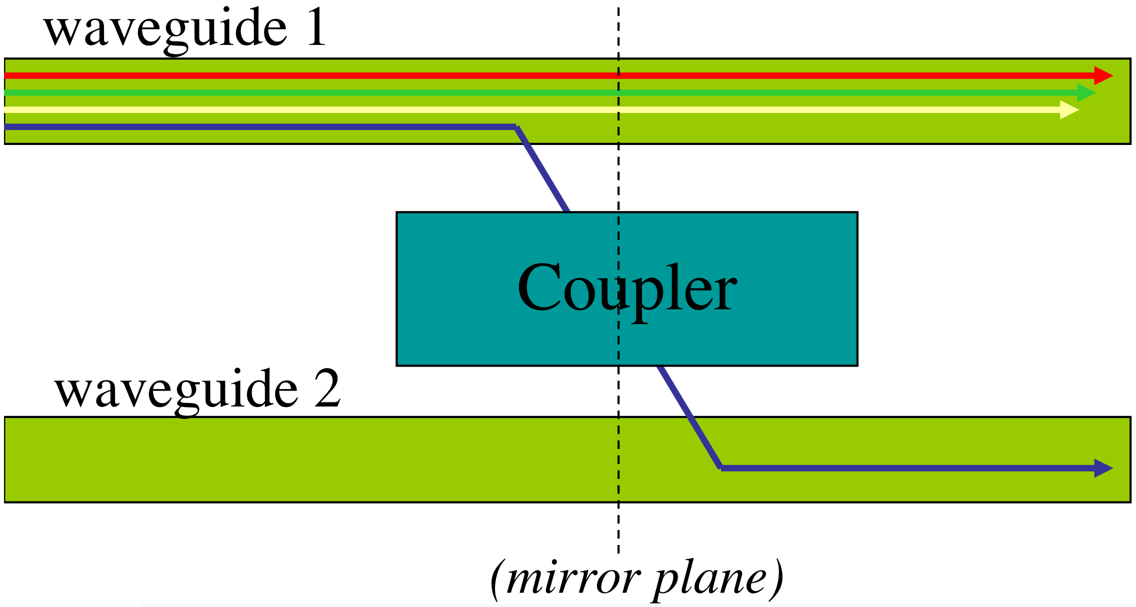
Resonant LED emission
luminus.com



(narrow-band) resonant absorption in a thin-film photovoltaic

[e.g. Ghebrebrhan (2009)]

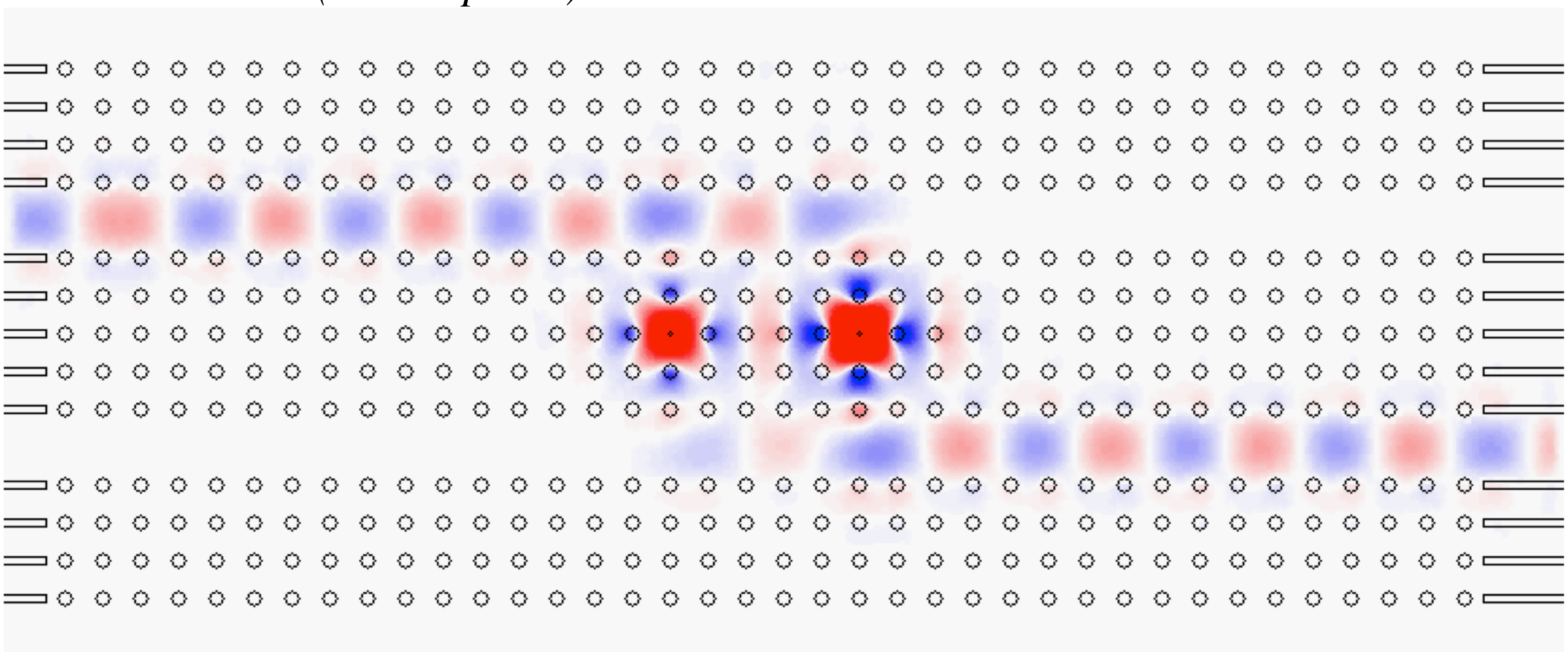
Another interesting example: Channel-Drop Filters



Perfect channel-dropping if:

Two resonant modes with:

- even and odd symmetry
- equal frequency (degenerate)
- equal decay rates



[S. Fan *et al*., *Phys. Rev. Lett*. **80**, 960 (1998)]

Dimensionless Losses: Q

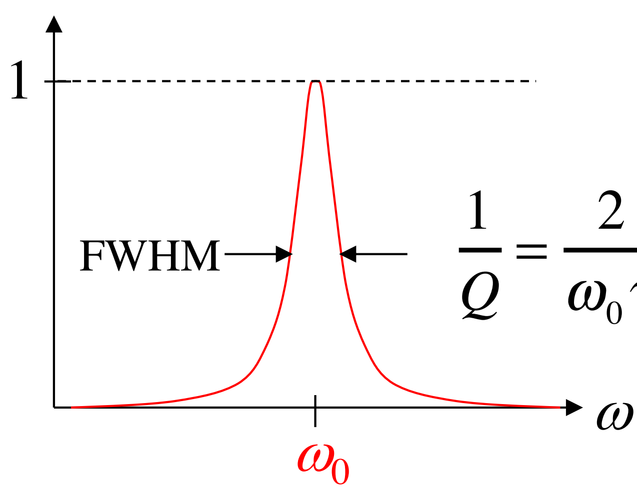
$$Q = \omega_0 \tau / 2$$

quality factor Q = # optical periods for energy to decay by $\exp(-2\pi)$

energy ~ $\exp(-\omega_0 t/Q) = \exp(-2t/\tau)$

in frequency domain: $1/Q$ = bandwidth

from temporal coupled-mode theory:



$$\frac{1}{Q} = \frac{2}{\omega_0 \tau}$$

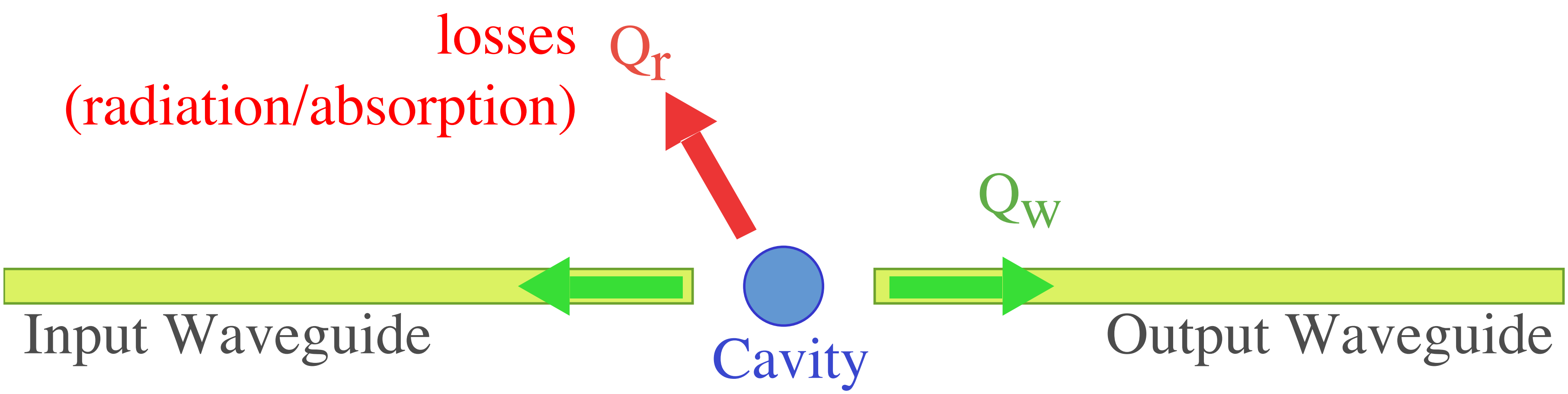
T = Lorentzian filter

$$= \frac{\frac{4}{\tau^2}}{(\omega - \omega_0)^2 + \frac{4}{\tau^2}}$$

…quality factor Q

More than one Q…

A simple model device (filters, bends, …):



$$\frac{1}{Q} = \frac{1}{Q_r} + \frac{1}{Q_w}$$

Q = lifetime/period

= frequency/bandwidth

We want: $Q_r \gg Q_w$

TCMT ⇒

$1 - \text{transmission} \sim 2Q / Q_r$

***worst case*:** high-Q (narrow-band) cavities

Nonlinearities + Microcavities?

weak effects

$\Delta n < 1\%$

very intense fields

& sensitive to small changes

A simple idea:

for the same input power, nonlinear effects are stronger in a microcavity

That's not all!

nonlinearities + microcavities

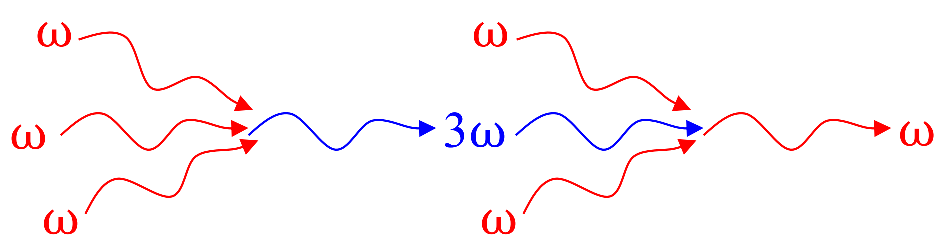
= *qualitatively* new phenomena

Nonlinear Optics

Kerr nonlinearities $\chi^{(3)}$: *(polarization ~ E^3)*

- Self-Phase Modulation (SPM)
  = change in refractive index(ω) ~ $|\mathbf{E}(\omega)|^2$
- Cross-Phase Modulation (XPM)
  = change in refractive index(ω) ~ $|\mathbf{E}(\omega_2)|^2$

- Third-Harmonic Generation (THG) & down-conversion (FWM)
  = $\omega \rightarrow 3\omega$, and back
- etc…



Second-order nonlinearities $\chi^{(2)}$: *(polarization ~ E^2)*

- Second-Harmonic Generation (SHG) & down-conversion
  = $\omega \rightarrow 2\omega$, and back
- Difference-Frequency Generation (DFG) = $\omega_1, \omega_2 \rightarrow \omega_1 - \omega_2$
- etc…

Nonlinearities + Microcavities?

weak effects

$\Delta n < 1\%$

very intense fields

& sensitive to small changes

A simple idea:

for the same input power, nonlinear effects are stronger in a microcavity

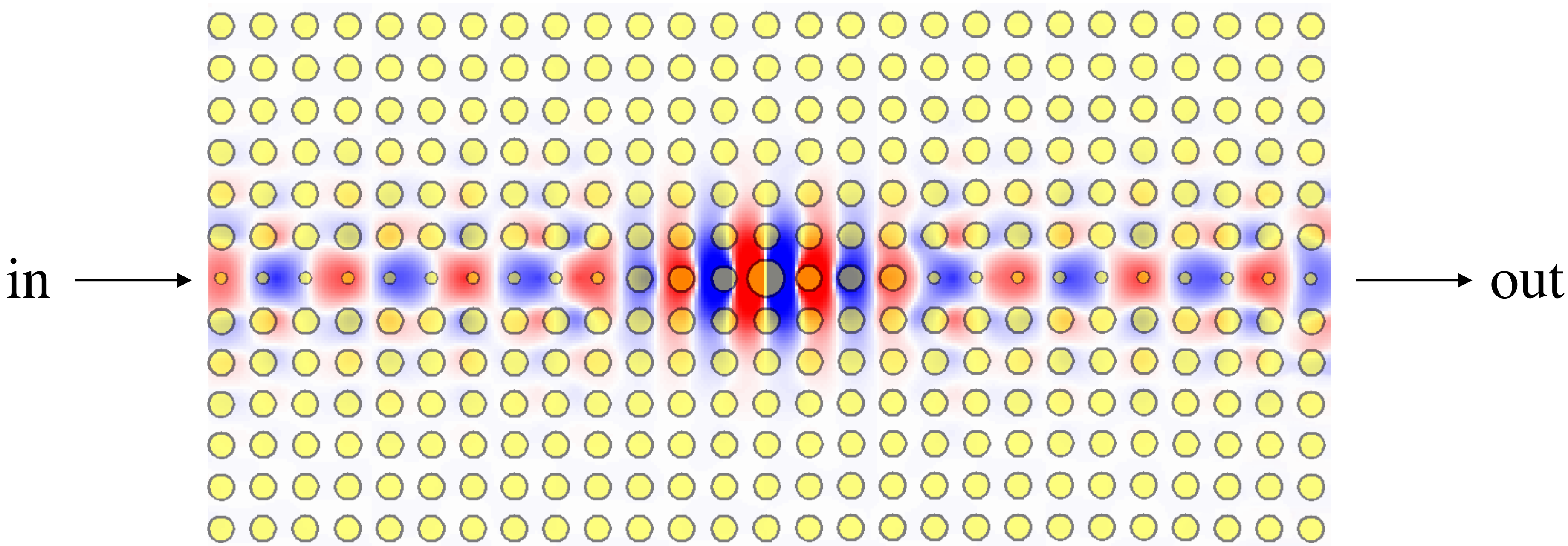
That's not all!

nonlinearities + microcavities

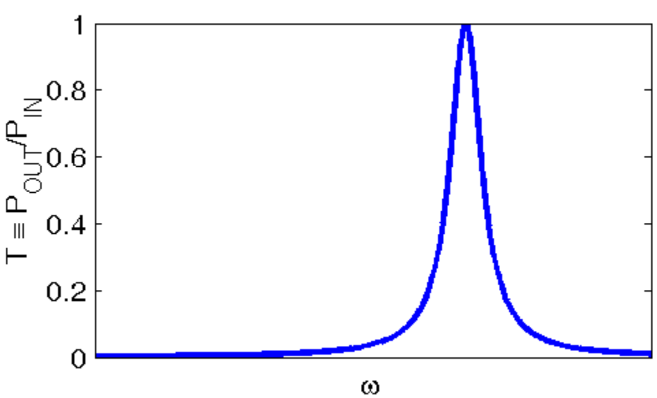
= *qualitatively* new phenomena

let's start with a well-known example from 1970's…

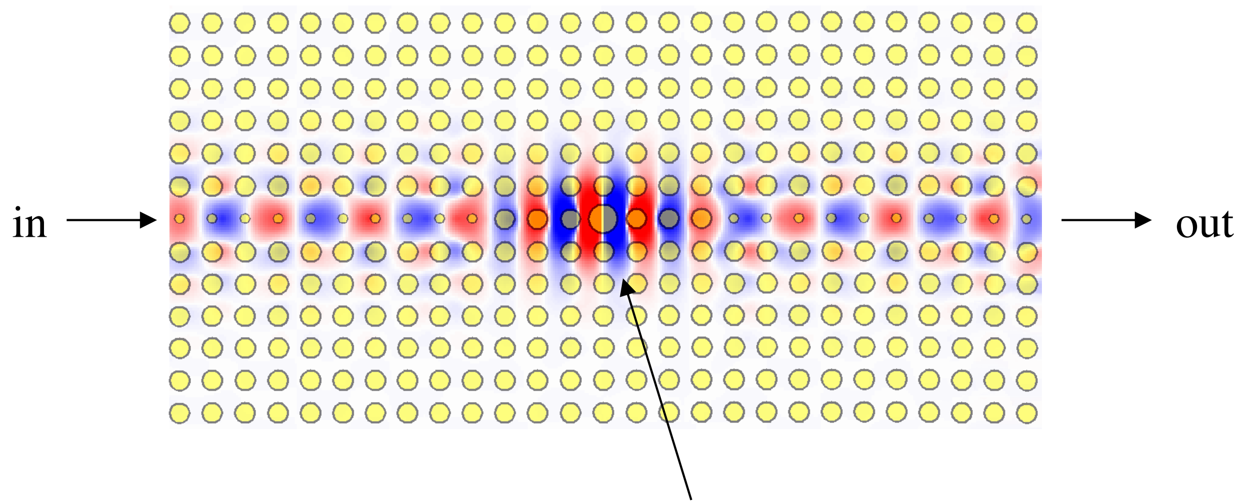
A Simple Linear Filter



Linear response:
Lorenzian Transmisson

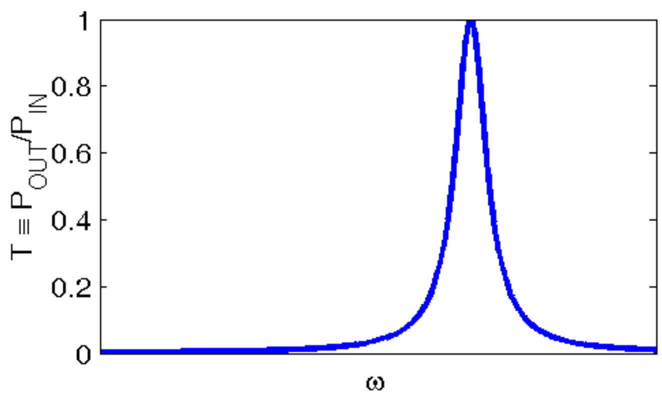


Filter + Kerr Nonlinearity?



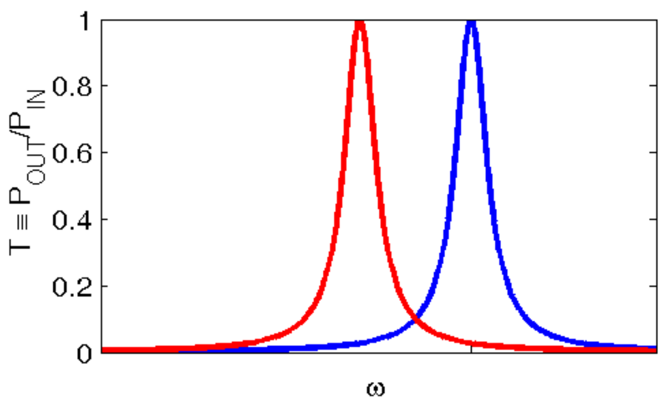
Kerr nonlinearity:
$\Delta n \sim |\mathbf{E}|^2$

Linear response:
Lorenzian Transmisson



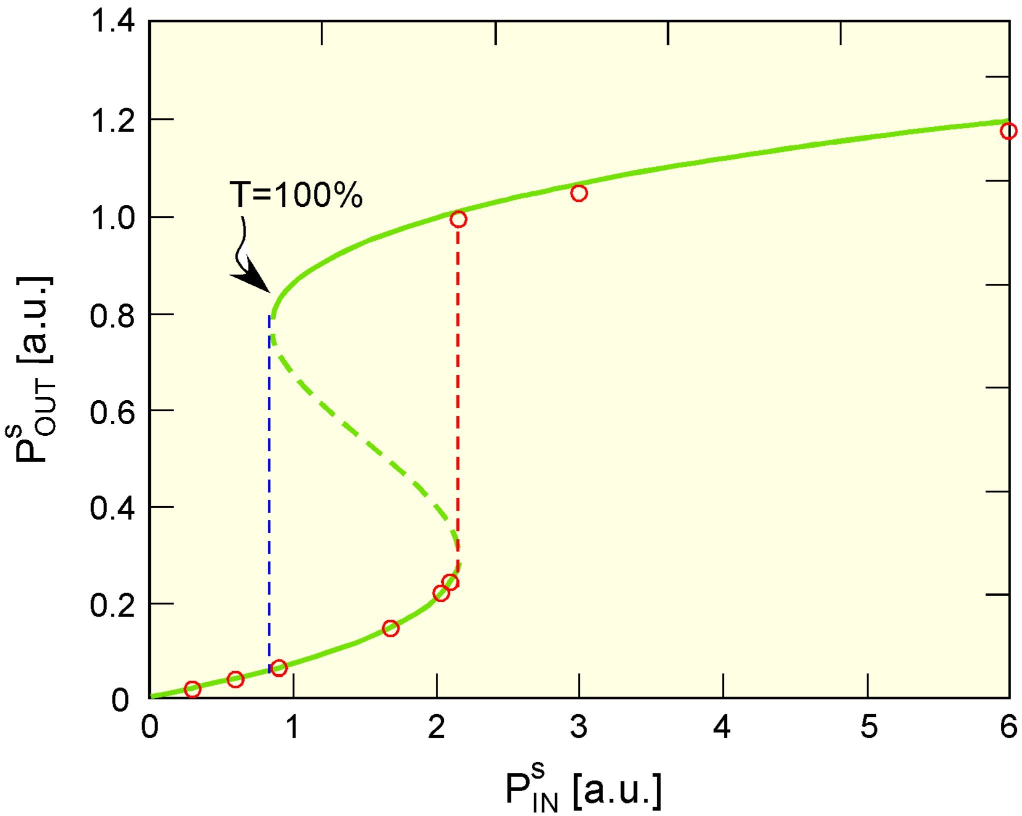
+ nonlinear
index shift
= ω shift

shifted peak?

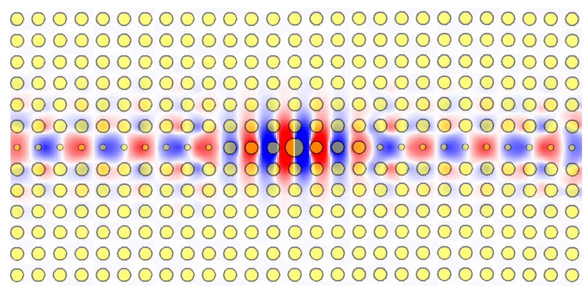


Optical Bistability

[Felber and Marburger., *Appl. Phys. Lett.* **28**, 731 (1978).]



Logic gates, switching, rectifiers, amplifiers, isolators, …



[Soljacic *et al.*, *PRE Rapid. Comm.* **66**, 055601 (2002).]

Bistable (hysteresis) response

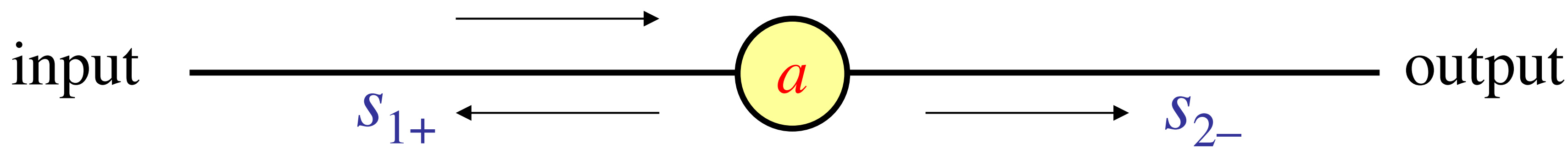
(& even multistable for multimode cavity)

Power threshold ~ V/Q^2

(in cavity with V ~ $(\lambda/2)^3$, for Si and telecom bandwidth power ~ mW)

TCMT for Bistability

[Soljacic *et al*., *PRE Rapid. Comm*. **66**, 055601 (2002).]



resonant cavity
frequency ω_0, lifetime τ,
SPM coefficient $\alpha \sim \chi^{(3)}$
(from perturbation theory)

$|s|^2$ = power

$|a|^2$ = energy

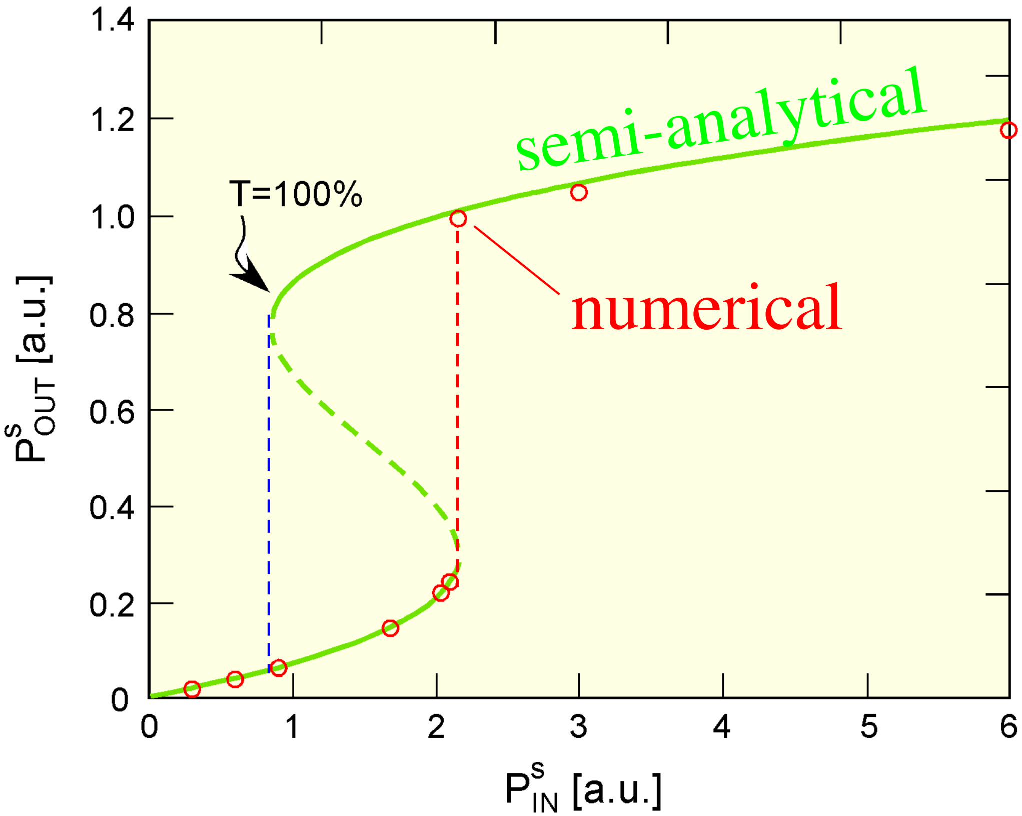
$$\frac{da}{dt} = -i(\omega_0 - \alpha|a|^2)a - \frac{2}{\tau}a + \sqrt{\frac{2}{\tau}}s_{1+}$$

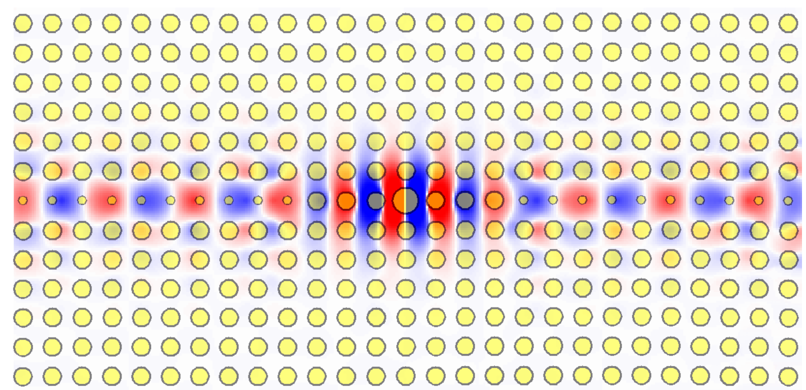
$$s_{1-} = -s_{1+} + \sqrt{\frac{2}{\tau}}a, \quad s_{2-} = \sqrt{\frac{2}{\tau}}a$$

gives cubic equation
for transmission
… bistable curve

Accuracy of Coupled-Mode Theory

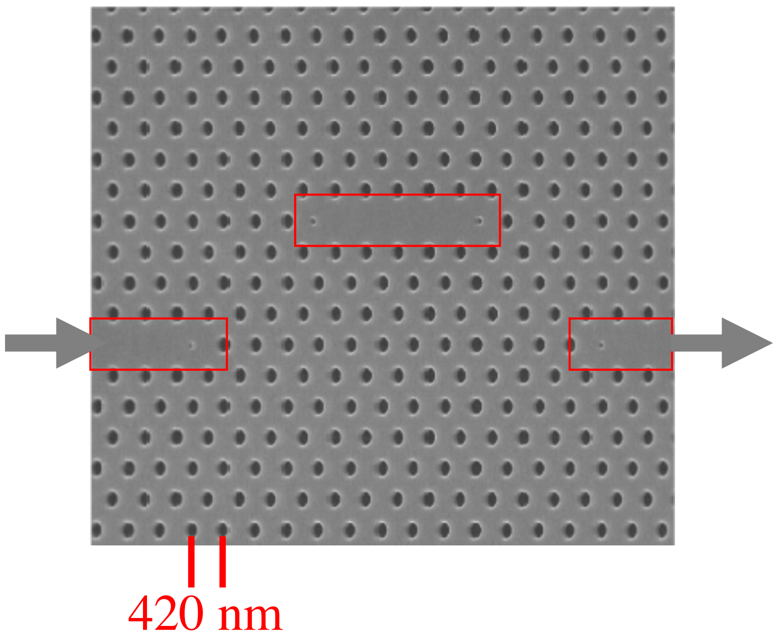
[Soljacic *et al*., *PRE Rapid*. *Comm*. **66**, 055601 (2002).]





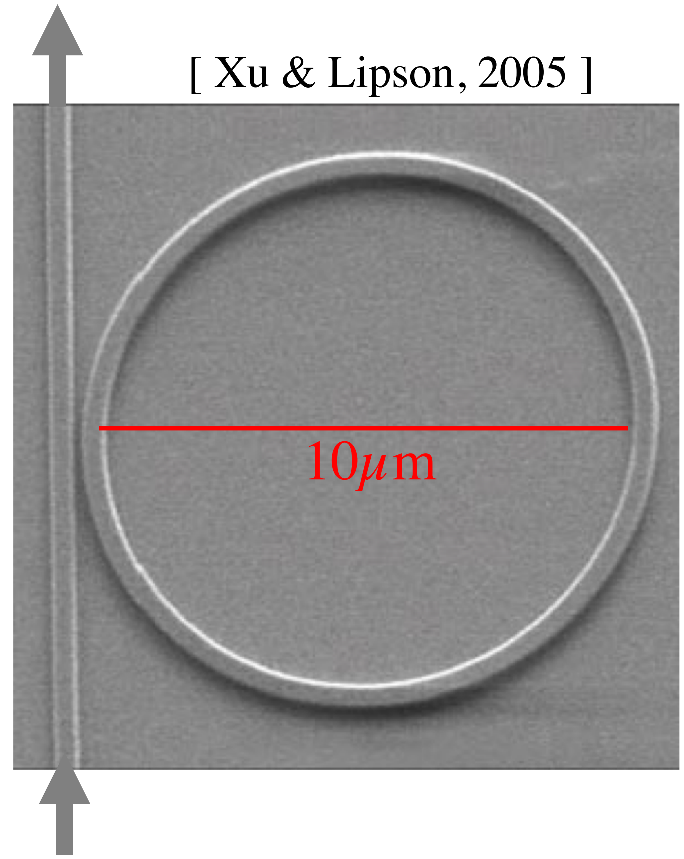
Optical Bistability in Practice

[Notomi *et al*. (2005).]



Q ~ 30,000
V ~ 10 optimum
Power threshold ~ 40 μW

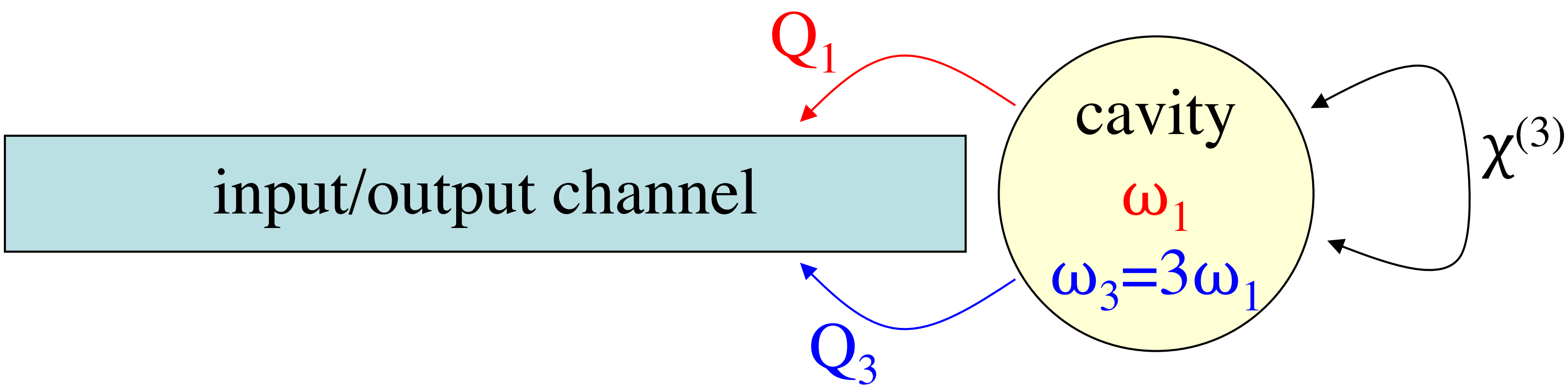
[Xu & Lipson, 2005]

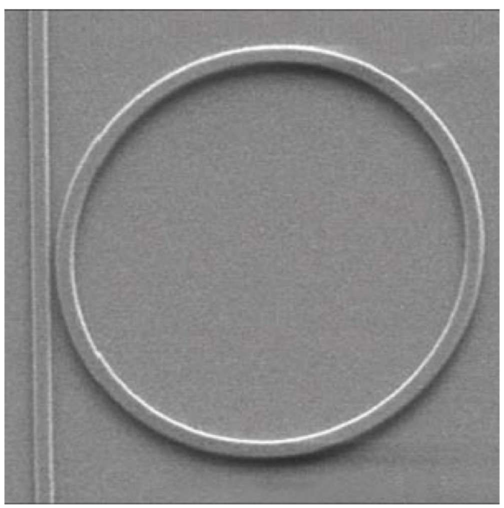


Q ~ 10,000
V ~ 300 optimum
Power threshold ~ 10 mW

THG in Doubly-Resonant Cavities

[publications from our group: H. Hashemi (2008) & A. Rodriguez (2007)]





e.g. ring resonator
with proper geometry

Not easy to make at micro-scale

— must precisely tune ω_3 / ω_1

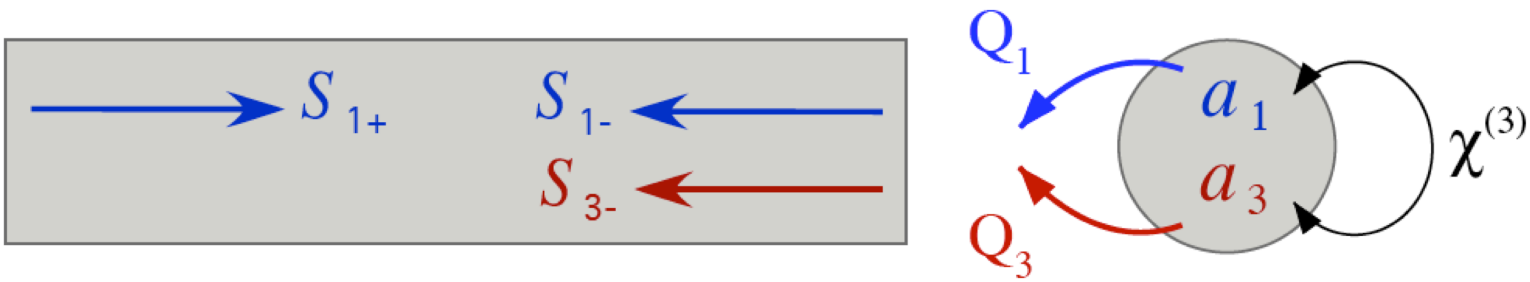
— materials must be ok at ω_1 and $3\omega_1$

But … what if we could do it?

… what are the consequences?

Coupled-mode Theory for THG

third harmonic generation



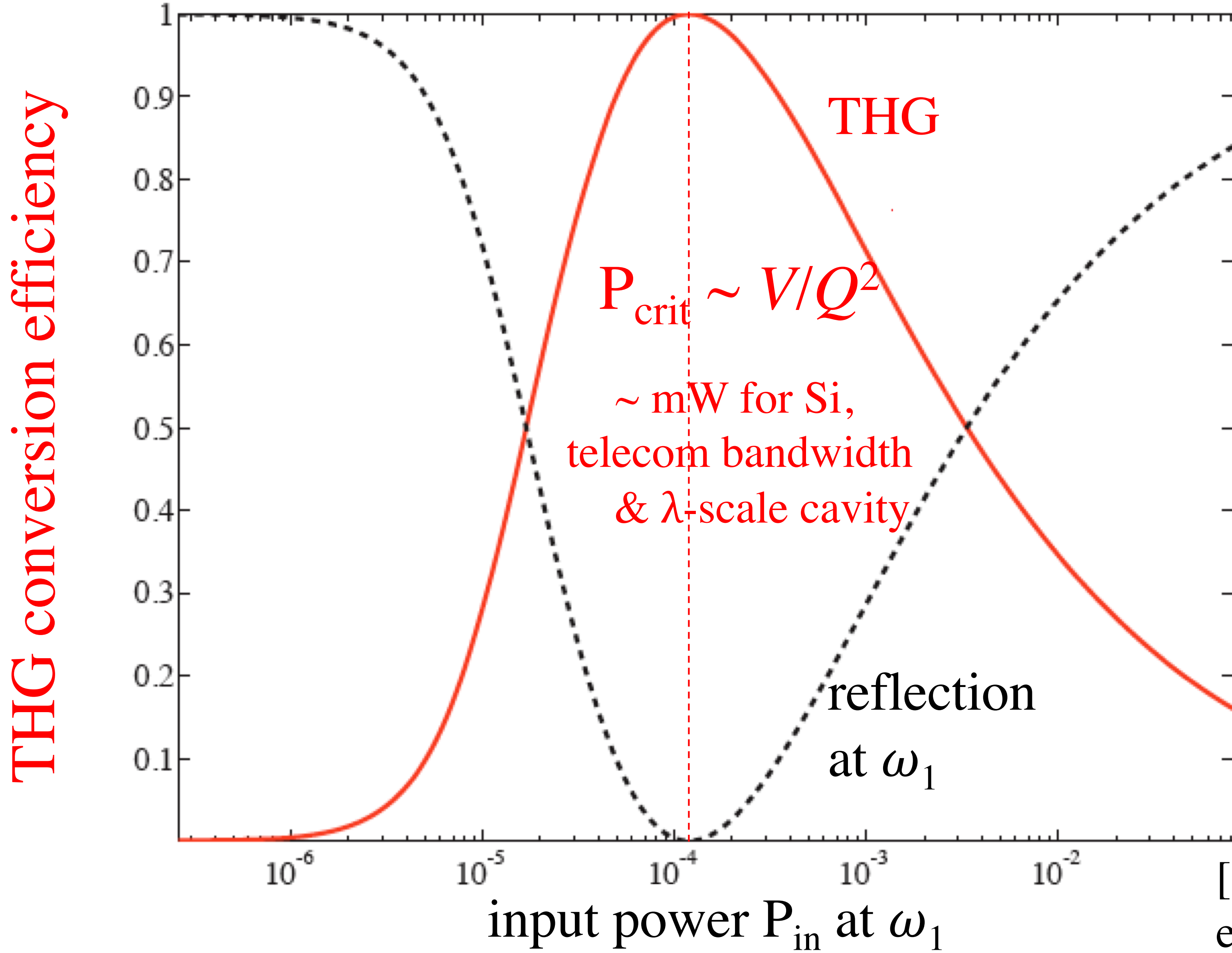
$$\frac{da_1}{dt} = \left(i\omega_1 \left(1 - \underbrace{\alpha_{11}|a_1|^2}_{\text{SPM}} - \underbrace{\alpha_{13}|a_3|^2}_{\text{XPM}}\right) - \frac{1}{\tau_1}\right) a_1 \underbrace{- i\omega_1\beta_1 (a_1^*)^2 a_3}_{\text{down-conversion}} + \sqrt{\frac{2}{\tau_{s,1}}} s_+$$

$$\frac{da_3}{dt} = \left(i\omega_3 \left(1 - \underbrace{\alpha_{33}|a_3|^2}_{\text{SPM}} - \underbrace{\alpha_{31}|a_1|^2}_{\text{XPM}}\right) - \frac{1}{\tau_3}\right) a_3 \underbrace{- i\omega_3\beta_3 a_1^3}_{\text{THG}} + \sqrt{\frac{2}{\tau_{s,3}}} s_+$$

[Rodriguez et al. (2007)]

α=0: Critical Power for Efficient THG

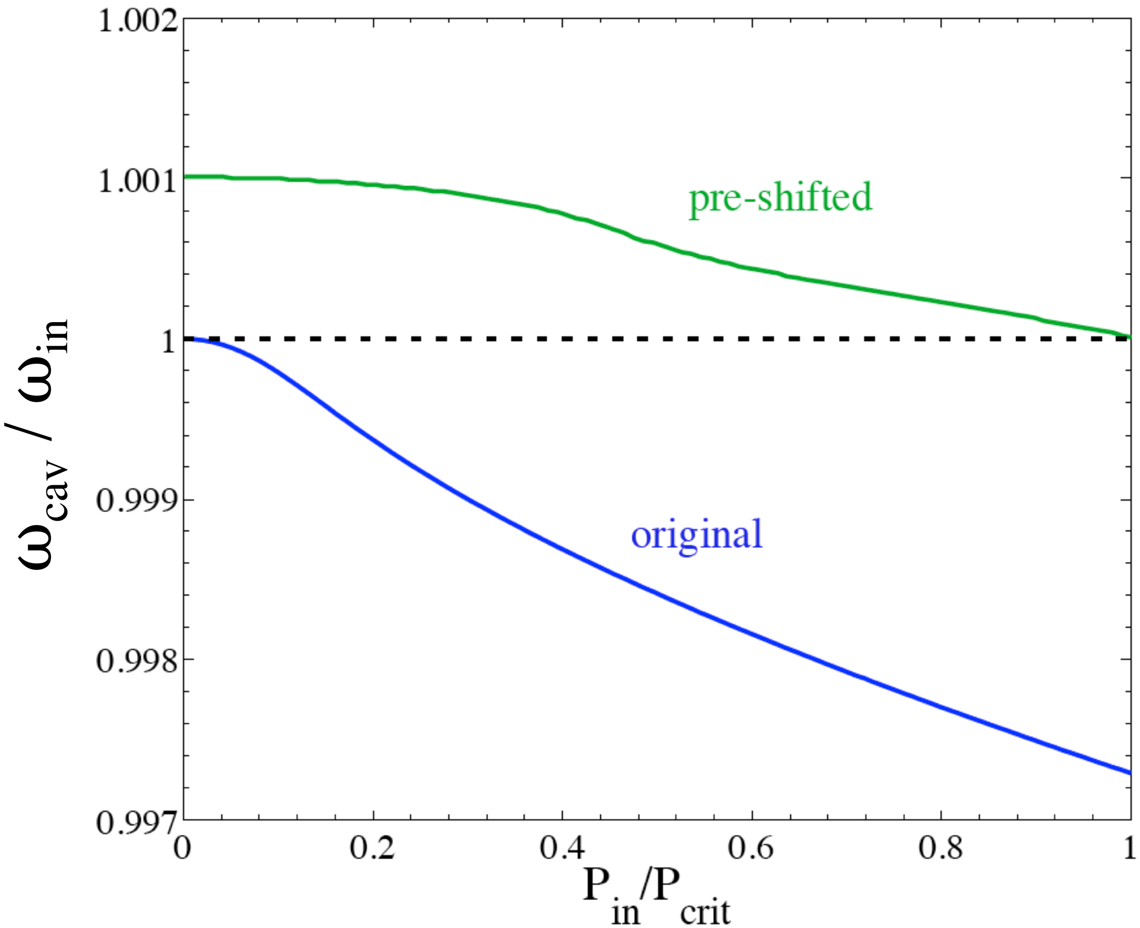
third-harmonic generation in doubly-resonant $\chi^{(3)}$ (Kerr) cavity



[Rodriguez et al. (2007)]

Detuning for Kerr THG

[Hashemi et al (2008)]



because of SPM/XPM, the input power changes resonant ω

…

compensate by pre-shifting resonance so that at $P_{in} = P_{crit}$ we have $\omega_3 = 3\ \omega_1$

Stability and Dynamics?

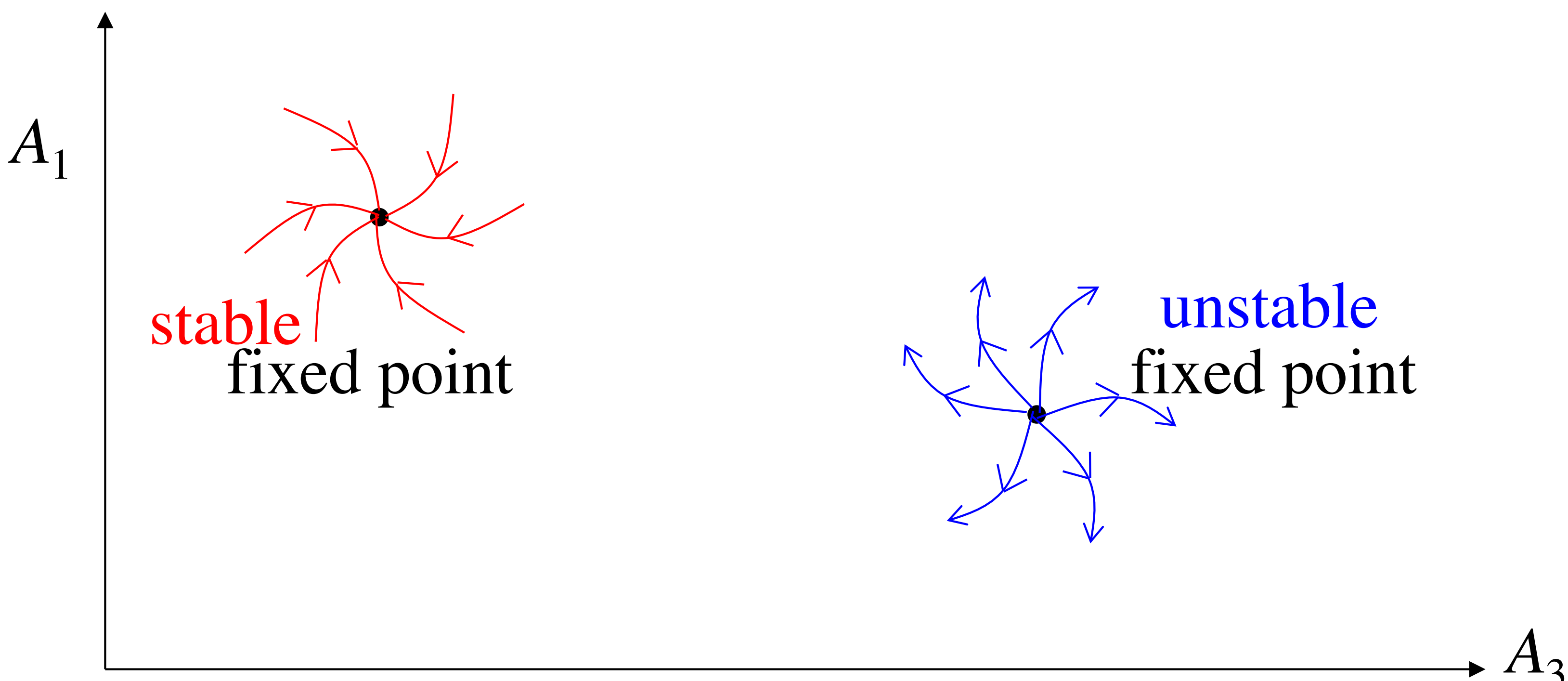
brief review

Steady state-solution: a_1 oscillating at ω_1, a_3 at ω_3

— rewrite equations in terms of $A_1 = a_1\, e^{i\omega_1 t}$

$$A_3 = a_3\, e^{i\omega_3 t}$$

then steady state = A_1, A_3 *constant* = fixed-point

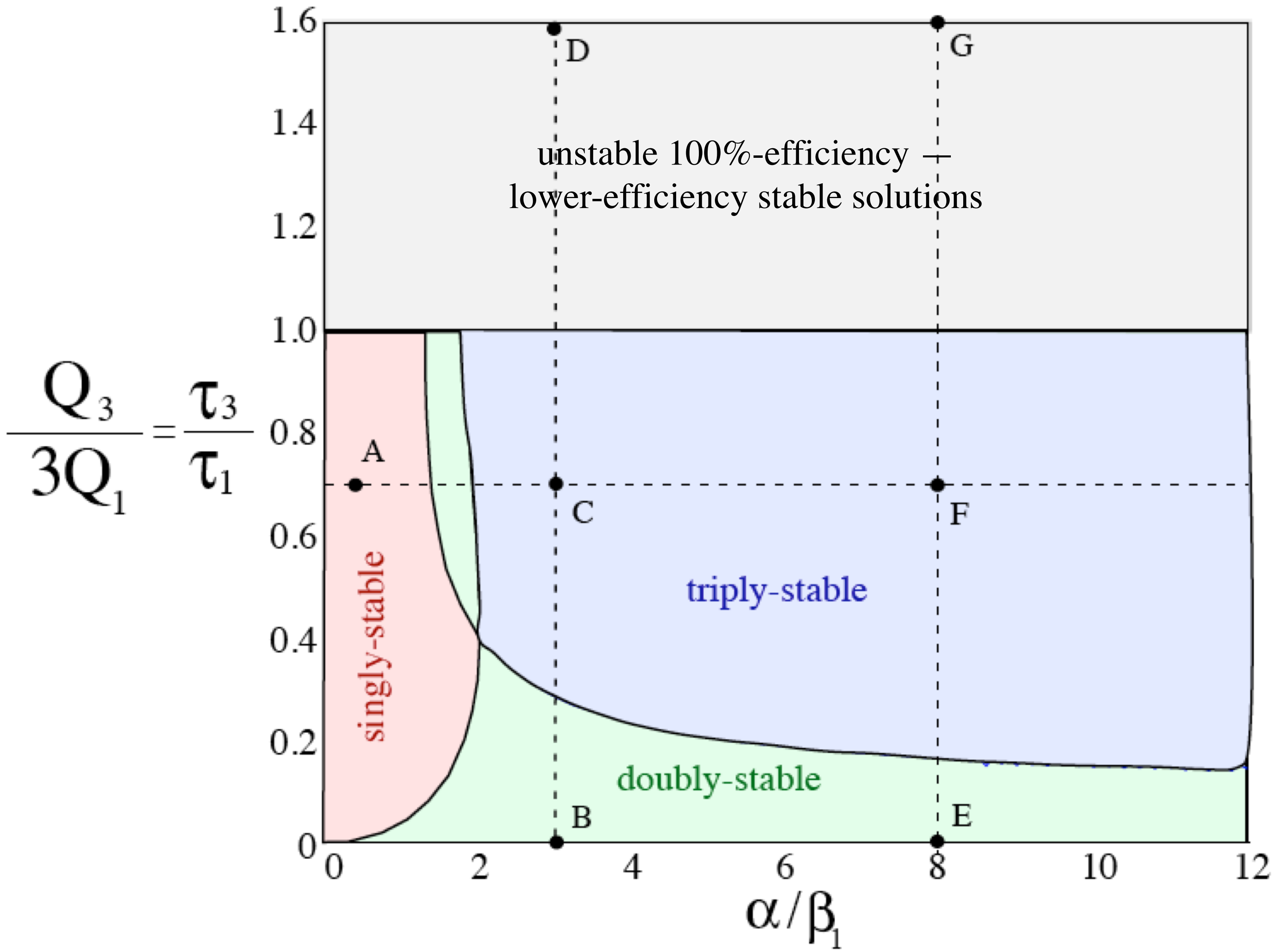


cartoon phase space (A_1, A_3 are actually complex) *for simplicity*, assume SPM = XPM coefficients:

$$\alpha_{11} = \alpha_{33} = \alpha_{13} = \alpha_{31} = \alpha$$

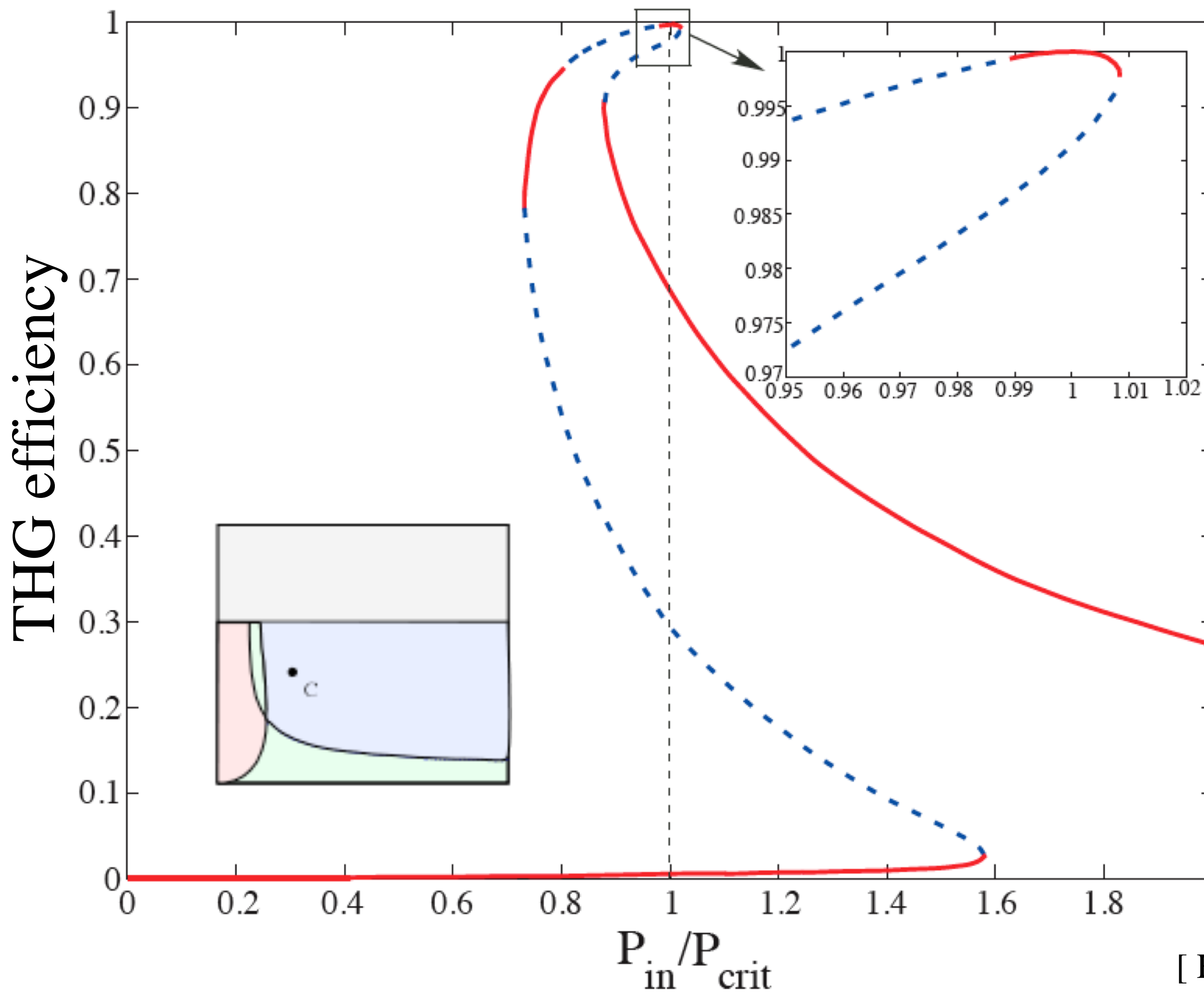
THG Stability Phase Diagram

[Hashemi et al (2008)]



SPM+XPM / THG

Bifurcation with Input Power



[Hashemi et al (2008)]

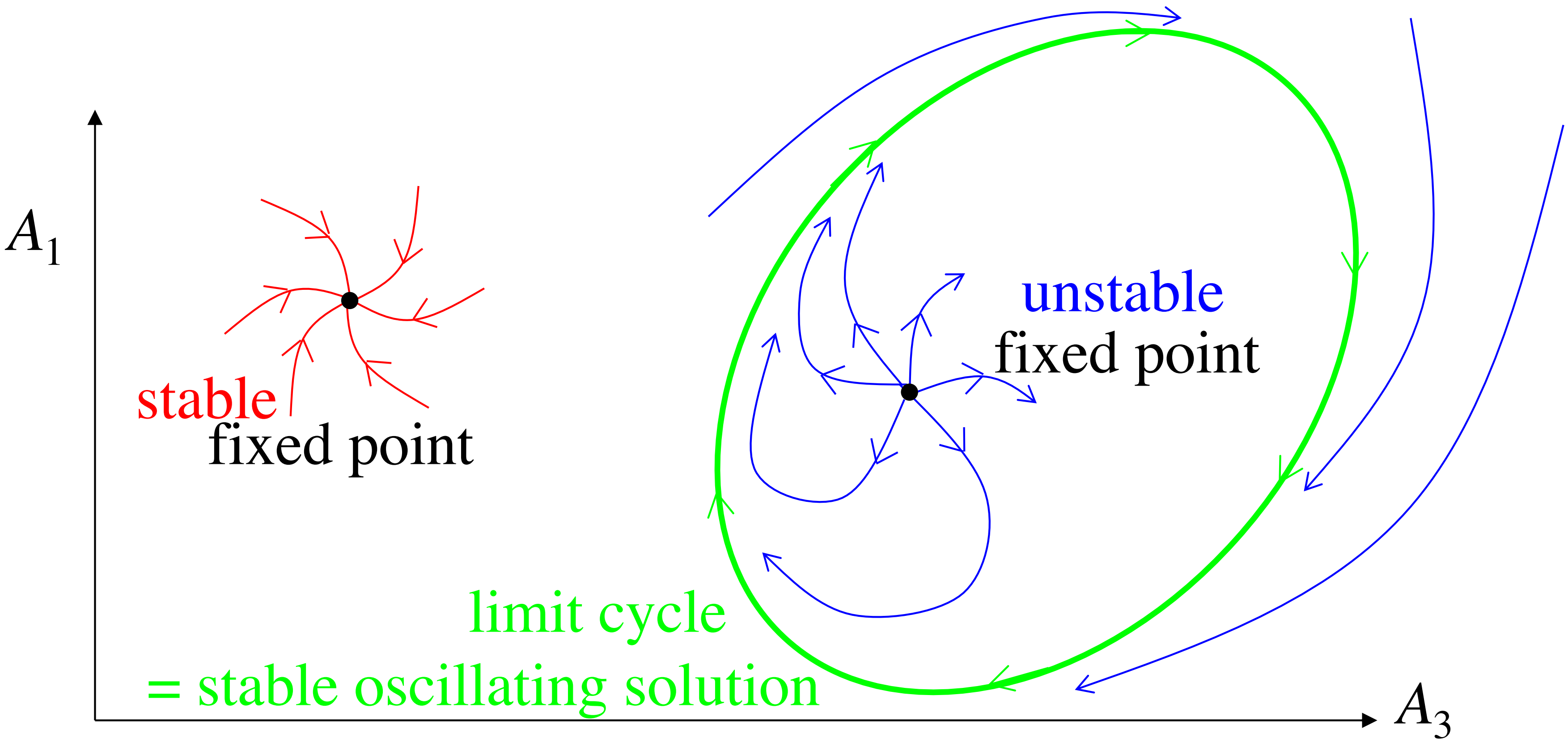
Limit Cycles

Steady state-solution: a_1 oscillating at ω_1, a_3 at ω_3

— rewrite equations in terms of $A_1 = a_1\, e^{i\omega_1 t}$

$A_3 = a_3\, e^{i\omega_3 t}$

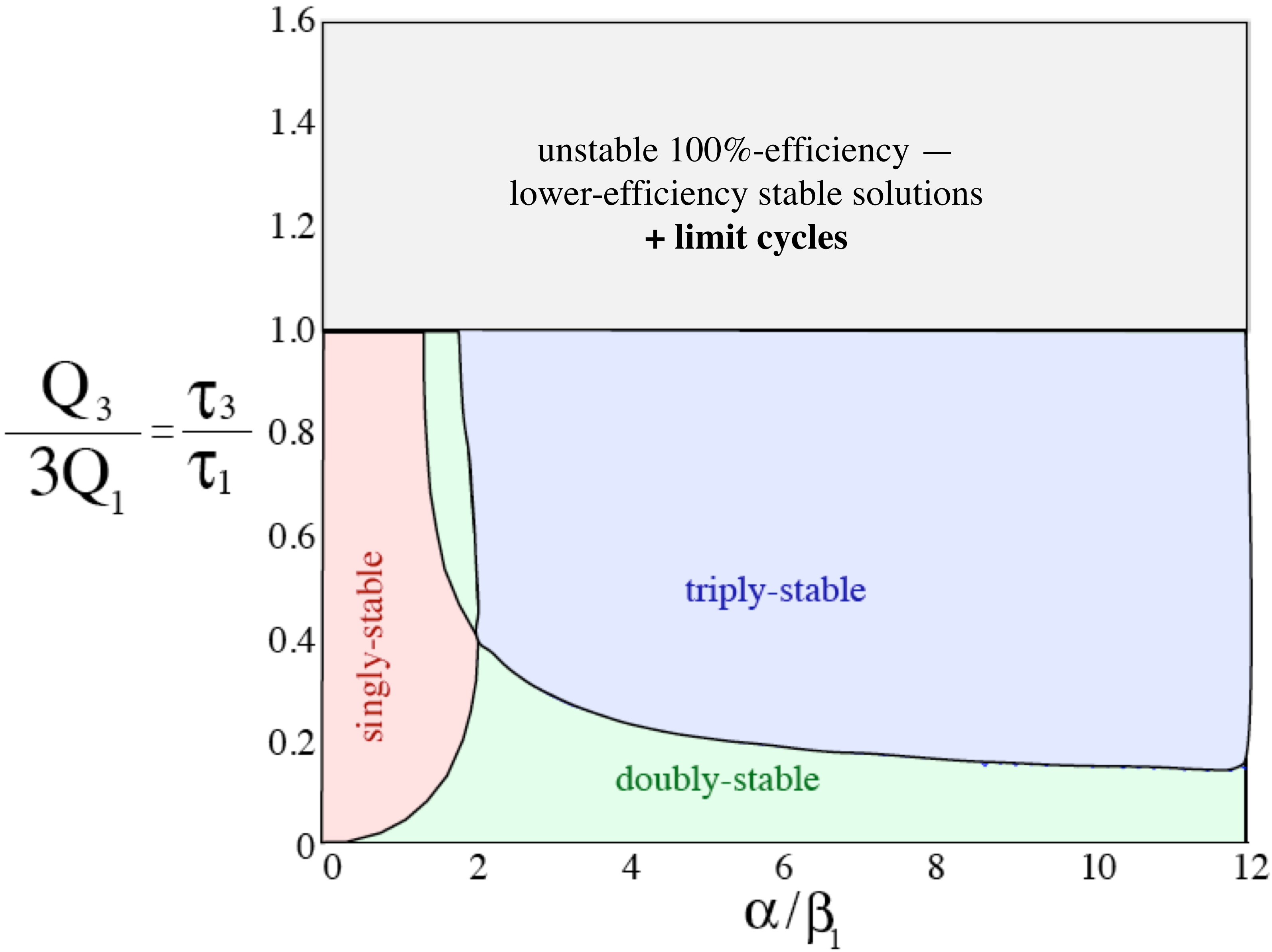
then steady state = A_1, A_3 *constant* = fixed-point



cartoon phase space (A_1, A_3 are actually complex)

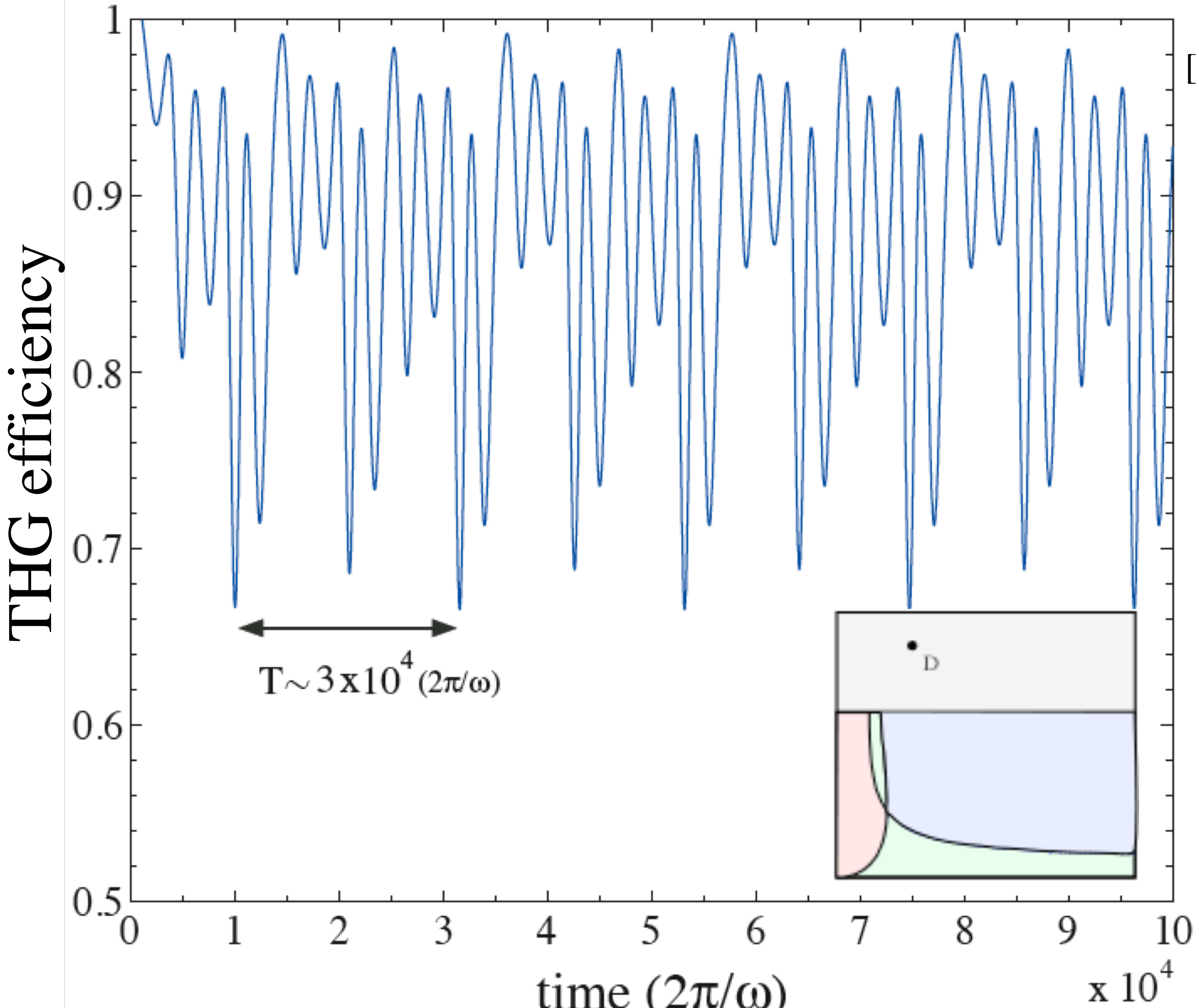
Stability Phase Diagram

[Hashemi et al (2008)]



An Optical Kerr-THG Oscillator

[analogous to self-pulsing in SHG; Drummond (1980)]



[Hashemi et al (2008)]