

## 18.335 Problem Set 4

Due Monday, 30 March 2015.

### Problem 1: Q's 'R us

- (a) Trefethen, problem 27.5
- (b) Trefethen, problem 28.2

### Problem 2: Power method

Suppose  $A$  is a diagonalizable matrix with eigenvectors  $\mathbf{v}_k$  and eigenvalues  $\lambda_k$ , in decreasing order  $|\lambda_1| \geq |\lambda_2| \geq \dots$ . Recall that the power method starts with a random  $\mathbf{x}$  and repeatedly computes  $\mathbf{x} \leftarrow A\mathbf{x}/\|A\mathbf{x}\|_2$ .

- (a) Suppose  $|\lambda_1| = |\lambda_2| > |\lambda_3|$ , but  $\lambda_1 \neq \lambda_2$ . Explain why the power method will not in general converge.
- (b) Give a *simple* fix to obtain  $\lambda_1$  and  $\lambda_2$  and  $\mathbf{v}_1$  and  $\mathbf{v}_2$  from the power method or some small modification thereof. (No fair going to some much more complicated/expensive algorithm like inverse iteration, Arnoldi, QR, or simultaneous iteration!) In particular, show that you can solve a  $2 \times 2$  Ritz-like eigenproblem to estimate both eigensolutions after enough power-method iterations.

### Problem 3: Arnoldi

Trefethen, problem 33.2.