

## 18.335 Problem Set 4

Due Monday, 28 October 2013.

### Problem 1: Q's 'R us

- (a) Trefethen, problem 27.5
- (b) Trefethen, problem 28.2

### Problem 2: Power method

Suppose  $A$  is a diagonalizable matrix with eigenvectors  $\mathbf{v}_k$  and eigenvalues  $\lambda_k$ , in decreasing order  $|\lambda_1| \geq |\lambda_2| \geq \dots$ . Recall that the power method starts with a random  $\mathbf{x}$  and repeatedly computes  $\mathbf{x} \leftarrow A\mathbf{x}/\|A\mathbf{x}\|_2$ .

- (a) Suppose  $|\lambda_1| = |\lambda_2| > |\lambda_3|$ , but  $\lambda_1 \neq \lambda_2$ . Explain why the power method will not in general converge.
- (b) Give a *simple* fix to obtain  $\lambda_1$  and  $\lambda_2$  and  $\mathbf{v}_1$  and  $\mathbf{v}_2$  from the power method or some small modification thereof. (No fair going to some much more complicated/expensive algorithm like inverse iteration, Arnoldi, QR, or simultaneous iteration!) In particular, show that you can solve a  $2 \times 2$  Ritz-like eigenproblem to estimate both eigensolutions after enough power-method iterations.

### Problem 3: Arnoldi

Trefethen, problem 33.2.

### Problem 4: GMRES

Trefethen, problem 35.4.

### Reminder: final project proposals

A half-page final-project proposal is **due on October 25**, outlining the goal and scope of your proposed paper—this is mainly so that I can give you feedback on whether your project is reasonable. Problems motivated by your research are perfectly fine, although you shouldn't simply recycle something you've already done, and I am mainly looking for *review* papers (of *published* results) rather than *research* papers (unpublished results). The only restriction is that, since PDEs are covered in 18.336 and other courses, I don't want projects where the primary focus is how to discretize the PDE (e.g. no projects

on discontinuous Galerkin methods or stable time-stepping, please). It is fine to take a discretized PDE as *input*, however, and then work on solvers, preconditioning, optimization, etcetera. Methods for ODEs are also fair game (especially recent developments that go beyond classic Runge-Kutta). One source of ideas might be to thumb through a copy of *Numerical Recipes* or a similar book and find a topic that interests you. Then go read some recent review papers on that topic (overview books like *Numerical Recipes* are not always trustworthy guides to a specific field).

You should **email** your final-project proposal to me; include *18.335 final project proposal* in the **subject** of the email. You can email the proposal to me early if you want! Your proposal should cite a couple of references that you will use as starting points.

See also the 18.335 web page, which answers some common questions about final projects.