18.335 Problem Set 4
Due Monday, 28 October 2013.

Problem 1: Q’s ‘R us
(a) Trefethen, problem 27.5
(b) Trefethen, problem 28.2

Problem 2: Power method
Suppose $A$ is a diagonalizable matrix with eigenvectors $v_k$ and eigenvalues $\lambda_k$, in decreasing order $|\lambda_1| \geq |\lambda_2| \geq \cdots$. Recall that the power method starts with a random $x$ and repeatedly computes $x \leftarrow Ax/\|Ax\|_2$.

(a) Suppose $|\lambda_1| = |\lambda_2| > |\lambda_3|$, but $\lambda_1 \neq \lambda_2$. Explain why the power method will not in general converge.

(b) Give a simple fix to obtain $\lambda_1$ and $\lambda_2$ and $v_1$ and $v_2$ from the power method or some small modification thereof. (No fair going to some much more complicated/expensive algorithm like inverse iteration, Arnoldi, QR, or simultaneous iteration!) In particular, show that you can solve a $2 \times 2$ Ritz-like eigenproblem to estimate both eigensolutions after enough power-method iterations.

Problem 3: Arnoldi
Trefethen, problem 33.2.

Problem 4: GMRES
Trefethen, problem 35.4.

Reminder: final project proposals
A half-page final-project proposal is due on October 25, outlining the goal and scope of your proposed paper—this is mainly so that I can give you feedback on whether your project is reasonable. Problems motivated by your research are perfectly fine, although you shouldn’t simply recycle something you’ve already done, and I am mainly looking for review papers (of published results) rather than research papers (unpublished results). The only restriction is that, since PDEs are covered in 18.336 and other courses, I don’t want projects where the primary focus is how to discretize the PDE (e.g. no projects on discontinuous Galerkin methods or stable time-stepping, please). It is fine to take a discretized PDE as input, however, and then work on solvers, preconditioning, optimization, etcetera. Methods for ODEs are also fair game (especially recent developments that go beyond classic Runge-Kutta). One source of ideas might be to thumb through a copy of Numerical Recipes or a similar book and find a topic that interests you. Then go read some recent review papers on that topic (overview books like Numerical Recipes are not always trustworthy guides to a specific field).

You should email your final-project proposal to me; include 18.335 final project proposal in the subject of the email. You can email the proposal to me early if you want! Your proposal should cite a couple of references that you will use as starting points.

See also the 18.335 web page, which answers some common questions about final projects.