18.335 Problem Set 5

Due Friday, 28 October 2011.

Problem 0:

Trefethen, problem 33.2.

Problem 1:

(a) Trefethen, 36.3. Plot the error in this eigenvalue as a function of how many Ax matrix-vector multiplies you perform (use a semilog or log-log scale as appropriate). (The files lanczos.m and A363.m posted on the web page are helpful.) Also plot the values of smallest *three* λ's versus the iteration, and show when ghost eigenvalues enter.

(b) Same problem, but use restarted Lanczos: after every 10 iterations of Lanzcos, restart with the best Ritz vector from those 10 iterations. Again, plot the error in λ_1 vs. matrix-vector multiply count.

(c) The above questions asked for the minimum-λ eigenvalue (which may be negative). Plot what happens if, instead, you try to get the minimum-$|\lambda|$ eigenvalue by these techniques. (Aside: a better way is to use Lanczos on A^{-1}, but that requires a fast way to solve $Ax = b$ in order to multiply by A^{-1}.)

Problem 2:

Trefethen, problem 38.6. (The files SD.m and A386.m on the web page are helpful.)

Problem 3:

In problem 3 of the Fall 2008 midterm for 18.335, it was claimed that you could use the conjugate-gradient algorithm for a Hermitian positive semidefinite matrix A, with a random starting guess, to find a vector in the null space (see the midterm solutions). Demonstrate this by means of an example, in Matlab, and plot the norm of the residual vs. iteration. (You can construct a random positive-semidefinite matrix A via, for example, `B=rand(198,200); A = B' * B`).