

# Options for solving $Ax=b$ ( $m \times m$ )

- If  $m$  is small ( $<10^4$ ), use LAPACK (LU, Cholesky, etc.)
- If  $m$  is moderate ( $<10^7$ ),  $A$  is sparse, and  $A$ 's sparsity comes from a mesh (especially 1d or 2d), consider a sparse-direct solver (UMFPACK, etc.)
- Otherwise, if  $m$  is large and  $Ax$  is fast:
  - if  $A$  is Hermitian positive-definite, use conjugate-gradient
  - if  $A$  is Hermitian indefinite:
    - if not too badly conditioned, use MINRES
    - otherwise, use SYMMLQ, GMRES, or some other scheme
  - if  $A$  is non-Hermitian, try several possibilities:
    - GMRES *if* convergence is achieved in  $\lesssim 100$  steps (e.g. you have a good preconditioner), otherwise some flavor of restarted GMRES
    - QMR (with look-ahead Lanczos)
    - BiCGSTAB(ell) with ell=1,2,4,... (ell>1 is most reliable)