18.335 Fall 2008
Performance Experiments with Matrix Multiplication

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Hardware: 2.66GHz Intel Core 2 Duo
64-bit mode, double precision, gcc 4.1.2

optimized BLAS dgemm: ATLAS 3.6.0
http://math-atlas.sourceforge.net/

A trivial problem?

$$\underset{m\times p}{C} = \underset{m\times n}{A}\ \underset{n\times p}{B}$$

the "obvious" C code:

```
/* C = A B, where A is m x n, B is n x p,
            and C is m x p, in row-major order */
void matmul(const double *A, const double *B,
            double *C, int m, int n, int p)
{
     int i, j, k;
     for (i = 0; i < m; ++i)
          for (j = 0; j < p; ++j) {
               double sum = 0;
               for (k = 0; k < n; ++k)
                    sum += A[i*n + k] * B[k*p + j];
               C[i*p + j] = sum;
          }
}
```

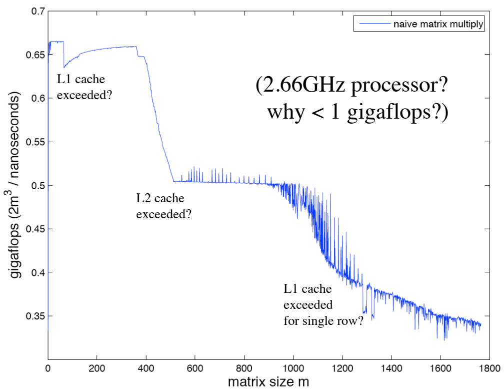
```
for $i$ = 1 to $m$
  for $j$ = 1 to $p$
    $C_{ij} = \sum_{k=1}^{n} A_{ik} B_{kj}$
```

$2mnp$ flops
(adds+mults)

just three loops, how complicated can it get?

flops/time is not constant!

(square matrices, m=n=p)



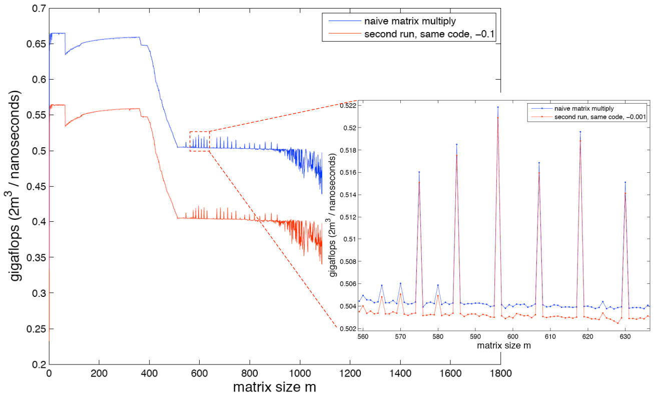
L1 cache exceeded?

(2.66GHz processor? why < 1 gigaflops?)

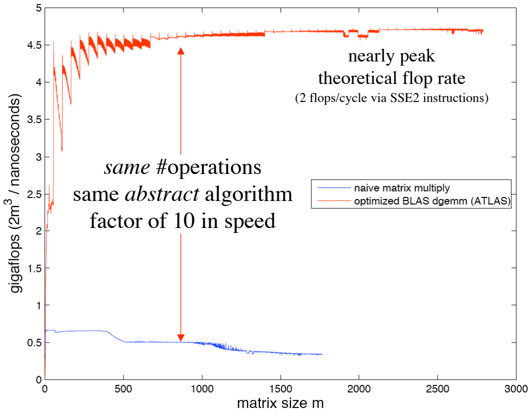
L2 cache exceeded?

L1 cache exceeded for single row?

Not all "noise" is random



All flops are not created equal



nearly peak theoretical flop rate
(2 flops/cycle via SSE2 instructions)

same #operations
same *abstract* algorithm
factor of 10 in speed

Things to remember

- We cannot understand performance without understanding memory efficiency (caches).
  - ~10 times more important than arithmetic count
- Computers are more complicated than you think.
- Even a trivial algorithm is nontrivial to implement *well*.
  - matrix multiplication: 10 lines of code → 130,000+ (ATLAS)