1. According to Coulomb’s law, the force between a particle of charge $q_1$ at the origin and a particle of charge $q_2$ at the point $\mathbf{r} = (x, y, z) \in \mathbb{R}^3$ is given by

$$
\mathbf{F} = \frac{q_1 q_2}{4\pi\varepsilon_0} \frac{\mathbf{r}}{|\mathbf{r}|^3},
$$

where $\varepsilon_0$ is a physical constant.

(a) Is $\mathbf{F}$ a conservative vector field? If so, find a function $\phi : \mathbb{R}^3 \to \mathbb{R}$ such that $\nabla \phi = \mathbf{F}$.

(b) If the distance between two charges is tripled, by what factor is the force between them reduced?

(c) How much work is required to move the second particle along the path

$$
\gamma(t) = (1 + (1 - t) \cos(t^2), \sqrt{\sin \pi t}, 4t - t^2) \quad 0 \leq t \leq 1?
$$

Express your answer in terms of $q_1$, $q_2$, and $\varepsilon_0$. 
2. (6.2.23 in Colley) Let $D$ be a region to which Green’s theorem applies and suppose that $u(x, y)$ and $v(x, y)$ are two functions of class $C^2$ whose domains include $D$. Show that 

$$\int\int_D \frac{\partial (u, v)}{\partial (x, y)} \, dA = \oint_C (u \nabla v) \cdot ds,$$

where $C = \partial D$ is oriented as in Green’s theorem.

3. (6.1.29 in Colley) Let $C$ be a level set of the function $f(x, y)$. Show that $\int_C \nabla f \cdot ds = 0.$