1. Evaluate \( \int_0^1 \int_0^{y^2} x^2 y \, dx \, dy \) and sketch the region of integration in \( \mathbb{R}^2 \) indicated by the limits of integration.

2. Evaluate \( \int_0^\pi \int_y^\pi \frac{\sin x}{x} \, dx \, dy \).
3. (Putnam exam '89) Evaluate \( \int_0^a \int_0^b e^{\max\{b^2x^2, a^2y^2\}} \, dy \, dx \) where \( a \) and \( b \) are positive.

4. (Fun/Challenge, based on 5.2.29 in Colley) Define a function \( f(x, y) \) on \([0, 1] \times [0, 2]\) by

\[
f(x, y) = \begin{cases} 
1 & \text{if } x \text{ is rational} \\
0 & \text{if } x \text{ is irrational and } y \leq 1 \\
2 & \text{if } x \text{ is irrational and } y > 1.
\end{cases}
\]

Show that the iterated Riemann integral \( \int_0^1 \int_0^2 f(x, y) \, dy \, dx \) exists, and find its value. Show that the iterated Riemann integral \( \int_0^2 \int_0^1 f(x, y) \, dx \, dy \) does not exist.