1. Let \( f(x, y) = e^{3x+y} \), and suppose that \( x = s^2 + t^2 \) and \( y = 2 + t \). Find \( \frac{\partial f}{\partial s} \) and \( \frac{\partial f}{\partial t} \) by substitution and by means of the chain rule. Verify that the results are the same for the two methods.

2. A conical ice sculpture melts in such a way that its height decreases at a rate of 0.001 meters per second and its radius decreases at a rate of 0.002 meters per second. At what rate is the volume of the sculpture decreasing when its height reaches 3 meters, assuming that its radius is 2 meters at that time? Express your answer in terms of \( \pi \) and in units of cubic meters per second.

3. Given a nonzero vector \( \mathbf{a} \in \mathbb{R}^n \), what unit vector \( \mathbf{u} \in \mathbb{R}^n \) maximizes the dot product \( \mathbf{a} \cdot \mathbf{u} \)? What unit vector minimizes the dot product? Prove that these really are the maximum and minimum, and comment on how this observation relates to the gradient \( \nabla f \) of a function \( f : \mathbb{R}^n \to \mathbb{R} \).
4. Consider the sphere $S$ passing through the point $P = (1,2,3)$ and centered at the origin. Find the equation of the plane tangent to $S$ at $P$.

5. Suppose $f : \mathbb{R}^2 \to \mathbb{R}$. Is it possible for $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ to exist at $(0,0)$ while $f$ is not differentiable at $(0,0)$? Prove that it isn't possible, or provide an example to show that it is possible.