1. Define $f : \mathbb{R}^3 \to \mathbb{R}$ by $f(x, y, z) = x^2 y + 2y + \sqrt{z} + 3$.

(a) Find $\nabla \times (\nabla f)$.

(b) Calculate $\int_C \nabla f \cdot ds$, where $C$ is the right half of the ellipse $2x^2 + y^2 = 1$ in the $x$-$y$ plane, oriented counterclockwise as shown below.

\begin{center}
\includegraphics[width=0.3\textwidth]{ellipse}
\end{center}

Solution. (a) The answer is $0$, because the curl of a gradient always vanishes.

(b) The answer is $f((0, 1, 0)) - f((0, -1, 0)) = 4$, because $\int_a^b \nabla f \cdot ds = f(b) - f(a)$.  \hfill $\square$