18.022 Recitation Quiz (with solutions)
17 November 2014

1. Consider the two parametrizations of the line segment \( S \) from \((0,0)\) to \((3,3)\) given by

\[
x_1(t) = (t, t) \quad 0 \leq t \leq 3 \\
x_2(t) = (2t, 2t) \quad 0 \leq t \leq 3/2.
\]

(a) Evaluate the line integral \( \int_S x^2 \, dx + y \, dy \) using the parametrization \( x_1 \).

(b) Evaluate the line integral \( \int_S x^2 \, dx + y \, dy \) using the parametrization \( x_2 \).

**Solution.** Using \( x_1 \), we get

\[
\int_0^3 t^2 \, dt + t \, dt = \left[ \frac{t^3}{3} + \frac{t^2}{2} \right]_0^3 = \frac{27}{2}
\]

Using \( x_2 \), we get

\[
\int_0^{3/2} (2t)^2 \, 2dt + (2t) \, 2dt = \left[ 4\frac{t^3}{3} + t^2 \right]_0^{3/2} = \frac{27}{2}
\]

This result is consistent with the parametrization-independence of the line integral. □