1. Show that if $S$ is a square in the plane with sides parallel to the axes and $A = \begin{pmatrix} 1 & \lambda \\ 0 & 1 \end{pmatrix}$, then $AS$ has the same area as $S$.

Solution. If $S$ has opposite corners at $(x, y)$ and $(x + h, y + h)$, then $A$ maps $S$ to the parallelogram with opposite corners at $(x + \lambda y, y)$ and $(x + h + \lambda(y + h), y + h)$. This parallelogram has a base of $(x + h + \lambda y - (x + \lambda y) = h$ and a height of $y + h - y = h$, so its area is $h^2$. Since the area of $S$ is also $h^2$, we conclude that $AS$ and $S$ have the same area. Note: $A$ is an example of a shear transformation.