1. Suppose that \( A = \begin{pmatrix} 4 & 0 \\ 0 & 3 \end{pmatrix} \) and \( B = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \). We regard \( A \) and \( B \) as maps from \( \mathbb{R}^2 \) to \( \mathbb{R}^2 \) by matrix multiplication (on the left, so \( A \) evaluated at \((1,2)\) is \((4,6)\), for example), and we denote by \( C \) the unit circle centered at the origin.

(a) Describe the image of \( C \) under the map \( AB \).

Solution. The matrix \( B \) rotates the plane 90 degrees counterclockwise. Therefore, the image of \( C \) under \( B \) is the unit circle rotated 90 degrees, which is equal to \( C \). The matrix \( A \) stretches the plane by a factor of 4 in the \( x \) direction and a factor of 3 in the \( y \) direction. Therefore, the image of \( C \) under \( AB \) is an ellipse centered at the origin with major axis of length 8 in the \( x \) direction and minor axis of length 6 in the \( y \) direction.

(b) Describe the image of \( C \) under the map \( BA \).

Solution. If we apply \( A \) first, then we get the ellipse described in the previous question. Rotating the ellipse 90 degrees counterclockwise gives the ellipse with major axis of length 8 in the \( y \) direction and minor axis of length 6 in the \( x \) direction.