Read all of Chapter 8.

1. Let $\text{SET-SPLITTING} = \{(S, C) | S$ is a finite set and $C = \{C_1, \ldots, C_k\}$ is a collection of subsets of $S$, where the elements of $S$ can be colored red or blue so every $C_i$ has at least one red element and at least one blue element}. Show that \text{SET-SPLITTING} is NP-complete.

2. For a cnf-formula $\phi$ with $m$ variables and $c$ clauses, show that you can construct in polynomial time an NFA with $O(cm)$ states that accepts all nonsatisfying assignments, represented as Boolean strings of length $m$. Conclude that $P \neq NP$ implies that NFAs cannot be minimized in polynomial time. Here, minimizing an NFA means finding an NFA with the fewest possible number of states that recognizes the same language as a given NFA.

3. Say that two Boolean formulas are equivalent if they have the same set of variables and are true on the same set of assignments to those variables (i.e., they describe the same Boolean function). A Boolean formula is minimal if no shorter Boolean formula is equivalent to it. (For definiteness, say that the length of a Boolean formula is the number of symbols it has.) Let $\text{MIN-FORMULA}$ be the collection of minimal Boolean formulas. Show that $\text{MIN-FORMULA} \in \text{PSPACE}$.

4. (a) Explain why the following argument fails to show that $\text{MIN-FORMULA} \in \text{coNP}$:
   i. If $\phi \notin \text{MIN-FORMULA}$, then $\phi$ has a smaller equivalent formula.
   ii. An NTM can verify that $\phi \in \text{MIN-FORMULA}$ by guessing that formula.

(b) Show (despite part a) that if $P = NP$, then $\text{MIN-FORMULA} \in \text{P}$.

5. For any positive integer $x$, let $x^R$ be the integer whose binary representation is the reverse of the binary representation of $x$. (Assume no leading 0s in the binary representation of $x$.) Define the function $R^+: \mathbb{N} \to \mathbb{N}$ where $R^+(x) = x + x^R$.

(a) Let $A_2 = \{(x, y) | R^+(x) = y\}$. Show $A_2 \in \text{L}$.

(b) Let $A_3 = \{(x, y) | R^+(R^+(x)) = y\}$. Show $A_3 \in \text{L}$.

6. Show that $A_{\text{NFA}}$ is NL-complete.

7. (optional) Let $B$ be the language of properly nested parentheses and brackets. For example, ([(())()[]]) is in $B$ but ([]) is not. Show that $B$ is in $L$. 
