Problem Set 5

Please turn in each problem on a separate page with your name.

Read all of Chapter 8.

1. Recall the language \( D = \{ \langle p \rangle \mid p \text{ is a polynomial in several variables having an integral root} \} \). We stated, but didn’t prove, that \( D \) is undecidable. In this problem, you are to prove a different property of \( D \)—namely, that \( D \) is \textit{NP-hard} if all problems in NP are polynomial time reducible to it, even though it may not be in NP itself. So you must show that all problems in NP are polynomial time reducible to \( D \).

2. Let \( \phi \) be a 3cnf-formula. An \( \neq \)-\textit{assignment} to the variables of \( \phi \) is one where each clause contains two literals with unequal truth values. In other words, an \( \neq \)-assignment satisfies \( \phi \) without assigning three true literals in any clause.

   (a) Show that the negation of any \( \neq \)-assignment to \( \phi \) is also an \( \neq \)-assignment.

   (b) Let \( \neq SAT \) be the collection of 3cnf-formulas that have an \( \neq \)-assignment. Show that we obtain a polynomial time reduction from 3SAT to \( \neq SAT \) by replacing each clause \( c_i \)

\[
(y_1 \lor y_2 \lor y_3)
\]

with the two clauses

\[
(y_1 \lor y_2 \lor z_i) \quad \text{and} \quad (\overline{z_i} \lor y_3 \lor b),
\]

where \( z_i \) is a new variable for each clause \( c_i \), and \( b \) is a single additional new variable.

(c) Conclude that \( \neq SAT \) is NP-complete.

3. Show that \( A_{LBA} = \{ \langle B, w \rangle \mid B \text{ is an LBA that accepts input } w \} \) is PSPACE-complete.

4. Show that \( E_{NFA} = \{ \langle A \rangle \mid A \text{ is an NFA and } L(A) = \emptyset \} \) is NL-complete.

5. For a cnf-formula \( \phi \) with \( m \) variables and \( c \) clauses, show that you can construct in polynomial time an NFA with \( O(cm) \) states that accepts all nonsatisfying assignments, represented as Boolean strings of length \( m \). Conclude that P \( \neq \) NP implies that NFAs cannot be minimized in polynomial time.

6. Say that two Boolean formulas are \textit{equivalent} if they have the same set of variables and are true on the same set of assignments to those variables (i.e., they describe the same Boolean function). A Boolean formula is \textit{minimal} if no shorter Boolean formula is equivalent to it. Let \( MIN-FORMULA \) be the collection of minimal Boolean formulas.

   (a) Show that \( MIN-FORMULA \in PSPACE \).

   (b) Show that if P = NP, then \( MIN-FORMULA \in P \).

7. \( \star \) (optional) Let \( B \) be the language of properly nested parentheses and brackets. For example, \( \langle (\langle \rangle) \rangle \rangle \) is in \( B \) but \( [\{\}][\}] \) is not. Show that \( B \) is in L.