18.404/6.840 Michael Sipser
Theory of Computation
Due in 2–108 by 11:00am sharp, Thursday, November 5, 2009

Problem Set 4

Please turn in each problem on a separate page with your name.

1. Book, 7.12 . [$MODEXP \in$ P]
2. Book, 7.17 . [if P = NP then nearly all in P are NP–complete]
3. Book, 7.27 . [*3COLOR* is NP–complete]
4. Book, 7.32 . [U is NP-complete]
   (The book has a typo. Should say NTM M instead of TM M.)
5. Book, 7.36 . [P = NP $\rightarrow$ can find assignment]
6. Let Σ be an alphabet and let $h\colon \Sigma \longrightarrow \Sigma$ be any function from Σ to itself. Extend h to a function from Σ^* to Σ^* in the obvious way, so that $h(w) = h(w_1)\cdots h(w_n)$ for $w_i \in \Sigma$. Further extend h to languages, so that $h(A) = \{h(w) |\ w \in A\}$.
   Demonstrate a language A in P and a function h, where $h(A)$ is NP-complete.
   (Hint: h is not necessarily a 1–1 function.)
7. * Let $A \subseteq 1^*$. Show that if A is NP-complete then P = NP.
   (Hint: Consider a reduction from SAT to A, and consider formulas such as ϕ_{0100} which set the first 4 variables x_1, x_2, x_3, x_4 in ϕ to the values $0, 1, 0, 0$ respectively.)