Problem Set 3

Please turn in each problem on a separate page with your name.

Read all of Chapter 5 and Section 6.1.

1. Consider the problem of determining whether a given Turing machine $M$ on a given input $w$ ever attempts to move its head left when its head is on the left-most tape cell. Formulate this problem as a language and show that it is undecidable by using a reduction from $A_{TM}$.

2. Let $A$ be a language.
   (a) Show that $A$ is Turing-recognizable iff $A \leq_m A_{TM}$.
   (b) Show that $A$ is decidable iff $A \leq_m 0^*1^*$.

3. Say that a variable $A$ in CFG $G$ is necessary if it appears in every derivation of some string $w \in G$. Let $NECESSARY_{CFG} = \{\langle G, A \rangle | A$ is a necessary variable in $G\}$.
   (a) Show that $NECESSARY_{CFG}$ is Turing-recognizable.
   (b) Show that $NECESSARY_{CFG}$ is undecidable.

4. Let $DISJOINT_{CFG} = \{\langle G, H \rangle | G$ and $H$ are CFGs and $L(G) \cap L(H) = \emptyset\}$. Show that $DISJOINT_{CFG}$ is undecidable. (Hint: Use a reduction from PCP. Given an instance $P = \{[t_1 b_1], [t_2 b_2], \ldots, [t_k b_k]\}$ of the Post Correspondence Problem, construct CFGs $G$ and $H$ with the rules:

   $G : T \rightarrow t_1 T a_1 | \cdots | t_k T a_k$
   $H : B \rightarrow b_1 B a_1 | \cdots | b_k B a_k$

   where $a_1, \ldots, a_k$ are new terminal symbols. Prove that this reduction works.)

5. Define a two-headed finite automaton (2DFA) to be a deterministic finite automaton that has two read-only, bidirectional heads that start at the left-hand end of the input tape and can be independently controlled to move in either direction. The tape of a 2DFA is finite and is just large enough to contain the input plus two additional blank tape cells, one on the left-hand end and one on the right-hand end, that serve as delimiters. A 2DFA accepts its input by entering a special accept state. For example, a 2DFA can recognize the language $\{a^n b^n c^n | n \geq 0\}$.
   (a) Let $A_{2DFA} = \{\langle M, x \rangle | M$ is a 2DFA and $M$ accepts $x\}$. Show that $A_{2DFA}$ is decidable.
   (b) Let $E_{2DFA} = \{\langle M \rangle | M$ is a 2DFA and $L(M) = \emptyset\}$. Show that $E_{2DFA}$ is not decidable.

6. Give an example in the spirit of the recursion theorem of a program in a real programming language (or a reasonable approximation of a programming language) that prints itself out.

7.* (optional) Show that $EQ_{TM} \not\leq_m EQ_{TM}$.

Midterm exam: Thursday, October 26, 2017, 2:30–4:00, top floor of Walker.
Covers Chapters 1, 2 (except 2.4), 3, 4, 5, and 6.1.

Final exam: Monday, December 18, 2017, 9:00–noon, duPont Gym.