Read all of Chapter 5 and Section 6.1.

0.1 Read and solve, but do not turn in: Book, 5.14 . [TM left end overrun is undecidable]

1. Consider the problem of determining whether a single-tape Turing machine ever writes a blank symbol over a nonblank symbol during the course of its computation on any input string. Formulate this problem as a language and show that it is undecidable.

2. Let $A$ be a language.
   
   (a) Show that $A$ is Turing-recognizable iff $A \leq_m A_{TM}$.
   
   (b) Show that $A$ is decidable iff $A \leq_m 0^*1^*$.

3. Let $AMBIG_{CFG} = \{ \langle G \rangle | G$ is an ambiguous CFG$\}$. Show that $AMBIG_{CFG}$ is undecidable.
   
   (Hint: Use a reduction from PCP. Given an instance $P = \{[t_1 b_1], [t_2 b_2], \ldots, [t_k b_k]\}$ of the Post Correspondence Problem, construct a CFG $G$ with the rules
   
   $$S \rightarrow T | B$$
   
   $$T \rightarrow t_1 Ta_1 | \cdots | t_k Ta_k | t_1 a_1 | \cdots | t_k a_k$$
   
   $$B \rightarrow b_1 Ba_1 | \cdots | b_k Ba_k | b_1 a_1 | \cdots | b_k a_k$$
   
   where $a_1, \ldots, a_k$ are new terminal symbols. Prove that this reduction works.)

4. Say that a variable $A$ in CFG $G$ is redundant if removing it and its associated rules leaves $L(G)$ unchanged. Let $REDUNDANT_{CFG} = \{ \langle G, A \rangle | A$ is a redundant variable in $G$\}.
   
   (a) Show that $REDUNDANT_{CFG}$ is Turing-recognizable.
   
   (b) Show that $REDUNDANT_{CFG}$ is undecidable.

5. Define a two-headed finite automaton (2DFA) to be a deterministic finite automaton that has two read-only, bidirectional heads that start at the left-hand end of the input tape and can be independently controlled to move in either direction. The tape of a 2DFA is finite and is just large enough to contain the input plus two additional blank tape cells, one on the left-hand end and one on the right-hand end, that serve as delimiters. A 2DFA accepts its input by entering a special accept state. For example, a 2DFA can recognize the language $\{a^n b^n c^n | n \geq 0\}$.
   
   (a) Let $A_{2DFA} = \{ \langle M, x \rangle | M$ is a 2DFA and $M$ accepts $x$\}. Show that $A_{2DFA}$ is decidable.
   
   (b) Let $E_{2DFA} = \{ \langle M \rangle | M$ is a 2DFA and $L(M) = \emptyset$\}. Show that $E_{2DFA}$ is not decidable.

6. Give an example in the spirit of the recursion theorem of a program in a real programming language (or a reasonable approximation of a programming language) that prints itself out.

7. (optional) Show that $EQ_{TM} \not\leq_m EQ_{TM}$.

Midterm exam: Thursday, October 15, 2020, 90 minutes, start time flexible. Covers Chapters 1, 2 (except 2.4), 3, 4, 5, and 6.1.

Final exam: Thursday, December 17, 2020, 3 hours, start time flexible. Covers the above plus Chapters 7, 8, 9.1, 9.2, 10.2 (except the section on primality), and 10.4 through Theorem 10.33.