Read all of Chapter 5 and Section 6.1.

0.1 Read and solve, but do not turn in: Book, 5.11. [TM second tape utilization is undecidable]

1. Consider the problem of determining whether a given Turing machine $M$ on a given input $w$ ever attempts to move its head left when its head is on the left-most tape cell. Formulate this problem as a language and show that it is undecidable by using a reduction from $A_{TM}$.

2. Let $AMBIG_{CFG} = \{ \langle G \rangle | G$ is an ambiguous CFG $\}$. Show that $AMBIG_{CFG}$ is undecidable. (Hint: Use a reduction from PCP. Given an instance $P = \{ \langle t_1 b_1 \rangle, \cdots, \langle t_k b_k \rangle \}$ of the Post Correspondence Problem, construct a CFG $G$ with the rules

\[
S \rightarrow T \mid B \\
T \rightarrow t_1 T a_1 \mid \cdots \mid t_k T a_k \mid t_1 a_1 \mid \cdots \mid t_k a_k \\
B \rightarrow b_1 B a_1 \mid \cdots \mid b_k B a_k \mid b_1 a_1 \mid \cdots \mid b_k a_k
\]

where $a_1, \ldots, a_k$ are new terminal symbols. Prove that this reduction works.)

3. Let $A$ be a language.

(a) Show that $A$ is Turing-recognizable iff $A \leq_m A_{TM}$.

(b) Show that $A$ is decidable iff $A \leq_m 0^*1^*$.

4. (a) Let $J = \{ w | \text{either } w = 0x \text{ for some } x \in A_{TM}, \text{ or } w = 1y \text{ for some } y \in \overline{A_{TM}} \}$. Use mapping reductions to show that neither $J$ nor $\overline{J}$ is Turing-recognizable.

(b) Let $FINITE_{TM} = \{ \langle T \rangle | T$ is a TM and $L(T)$ is a finite language $\}$. Show that $A_{TM} \leq_m FINITE_{TM}$ to prove that $FINITE_{TM}$ is not T-recognizable.

5. An erasing Turing machine ($eTM$) operates like an ordinary deterministic TM except that when its head reads a symbol on the tape, the $eTM$ may either leave that symbol unchanged or it may change it to the usual blank symbol ($\omega$). Changing a tape symbol to a nonblank symbol is prohibited. Formally speaking, the $eTM$ transition function $\delta$ may have $\delta(q, a) = (r, b, M)$ only where $b = a$ or $b = \omega$ for $q, r \in Q$ and $M \in \{ L, R \}$.

(a) Let $B = \{ \#a^k b^k \# | k \geq 0 \}$ and $\Sigma = \{ a, b, \# \}$. Briefly describe an $eTM$ that decides $B$.

(b) Let $E_{eTM} = \{ \langle B \rangle | B$ is an $eTM$ where $L(B) = \emptyset \}$. Prove that $E_{eTM}$ is undecidable. (Provide enough detail to show how your proof depends on the $eTM$ model.)

6. Give a Python program that prints itself out, in the spirit of the recursion theorem. If you don’t know Python, use some other programming language or an approximation of one. (Hint: Make a function that takes a string input. The substring operation $S[i:j]$ and escaped quotes are useful. A clean solution avoids implementation features such as character codes.)

7. (optional) Read section 6.3 on Turing reducibility ($\leq_T$). Turing reducibility formalizes the notion of “general reducibility” described in lecture. Show that $EQ_{TM} \not\leq_T A_{TM}$.

Midterm exam: Thursday, October 21, 2021, 2:30–4:00, top floor of Walker.
Covers Chapters 1, 2 (except 2.4), 3, 4, 5, and 6.1.

Final exam: Friday, December 17, 2021, 1:30–4:30, top floor of Walker.
Covers the above plus Chapters 7, 8, 9.1, 9.2, 10.2 (except the section on primality), and 10.4 through Theorem 10.33.