18.404/6.840 Michael Sipser
Theory of Computation
Due in 2–108 by 11:00am sharp, Thursday, October 8, 2009

Problem Set 2 – #7 revised

Please turn in each problem on a separate page with your name.

1. The language *GOINGUP* contains all strings with increasing runs of 1s separated by #s. Formally, $GOINGUP = \{w|\ w = \mathtt{1}^{n_1}\mathtt{\#1}^{n_2}\mathtt{\#1}^{n_3}\mathtt{\#}\cdots\mathtt{\#1}^{n_k}$, where $k \geq 0$ and $n_i \leq n_j$ if $i \leq j\}$. Here $\Sigma = \{\mathtt{\#}, \mathtt{1}\}$. Let $\overline{GOINGUP}$ be the complement of *GOINGUP*.

   (a) Give a CFG generating $\overline{GOINGUP}$.

   (b) Describe a PDA recognizing $\overline{GOINGUP}$.

2. Let *GOINGUP* be defined as in Problem 1. Prove that *GOINGUP* is not a CFL.
   (Note that Problems 1 and 2 together demonstrate that the class of CFLs isn't closed under either complement or intersection. Check that you understand why.)

3. Book, 3.14 . [queue automata]

4. Book, 3.18 . [decidable iff enumerable in lex order]

5. Book, 4.17 . [projection of decidable iff T-recognizable]

6. Say x is a ***substring*** of y if $uxv = y$ for some $u, v \in \Sigma^*$.
   Let $SUBSTRING_{\text{CFG}} = \{\langle G, x\rangle|\ G$ is a CFG and x is a substring of some $y \in L(G)\}$.
   Show that $SUBSTRING_{\text{CFG}}$ is decidable.

7.⋆ Let $\Sigma = \{\mathtt{0}, \mathtt{1}\}$ and let C_1 be the language of all strings that contain a 1 in their middle third. and let C_2 be the language of all strings that contain two 1s in their middle third.

   In other words $C_1 = \{xyz|\ x, z \in \Sigma^*$ and $y \in \Sigma^*\mathtt{1}\Sigma^*$, where $|x| = |z| \geq |y|\}$ and
   $C_2 = \{xyz|\ x, z \in \Sigma^*$ and $y \in \Sigma^*\mathtt{1}\Sigma^*\mathtt{1}\Sigma^*$, where $|x| = |z| \geq |y|\}$.

   (a) Show that C_1 is a CFL.

   (b) Show that C_2 is not a CFL.

8.⋆ Let the ***rotational closure*** of language A be $RC(A) = \{yx|\ xy \in A$ where $x, y \in \Sigma^*\}$.
   Show that the class of CFLs is closed inder rotational closure.