Problem Set 2

Please turn in each problem on a separate page with your name.

Read all of Chapters 3 and 4.

0.1 Read and solve, but do not turn in: Book, 2.16. [CFLs is closed under $\cup$, $\circ$, $\ast$]
Solve by using both CFLs and PDAs.

0.2 Read and solve, but do not turn in: Book, 2.18. [CFL $\cap$ regular = CFL]
You can check your solution with the one in the book.

0.3 Read and solve, but do not turn in: Book, 2.26. [Chomsky normal form]

1. Let $\Sigma = \{0, 1\}$ and let $C_2 = \{tut | t \in 0^* \text{ and } u \in 0^*10^*10^*, \text{where } |t| = |u|\}.$
   (a) Show that $C_2$ is not a CFL.
   (b) Is $C_2 \cup (\Sigma\Sigma)^*$ a CFL? Why or why not?
   (c) Is $C_2 \cup \Sigma(\Sigma\Sigma)^*$ a CFL? Why or why not?

2. Let $G = (V, \Sigma, R, \langle \text{stmt} \rangle)$ be the following grammar. $\Sigma = \{\text{if, condition, then, else, a:=1}\}$,
   $V = \{\langle \text{stmt} \rangle, \langle \text{if-then} \rangle, \langle \text{if-then-else} \rangle, \langle \text{assign} \rangle\}$ and the rules are:
   $$\langle \text{stmt} \rangle \rightarrow \langle \text{assign} \rangle \mid \langle \text{if-then} \rangle \mid \langle \text{if-then-else} \rangle$$
   $$\langle \text{if-then} \rangle \rightarrow \text{if condition then } \langle \text{stmt} \rangle$$
   $$\langle \text{if-then-else} \rangle \rightarrow \text{if condition then } \langle \text{stmt} \rangle \text{ else } \langle \text{stmt} \rangle$$
   $$\langle \text{assign} \rangle \rightarrow \text{a:=1}$$

   (a) Show that $G$ is ambiguous.
   (b) Give a new unambiguous grammar that generates $L(G)$.
      (You do not need to prove that your grammar works or that it is unambiguous, but
      please add a few comments about why it does work to help the grader.)

3. A Turing machine with left reset is similar to an ordinary Turing machine, but the
   transition function has the form $\delta: Q \times \Gamma \rightarrow Q \times \Gamma \times \{R, \text{RESET}\}.$ If $\delta(q,a) = (r,b,\text{RESET})$,
   when the machine is in state $q$ reading an $a$, the machine's head jumps to the left-hand end
   of the tape after it writes $b$ on the tape and enters state $r$. Note that these machines do not
   have the usual ability to move the head one symbol left. Show that Turing machines with
   left reset recognize the class of Turing-recognizable languages.

4. Let $PAL_{\text{DFA}} = \{\langle M \rangle | M \text{ is a DFA that accepts some palindrome}\}$. Show that $PAL_{\text{DFA}}$ is
   decidable. (Hint: Theorems from lecture are helpful here.)

5. Let $A$ and $B$ be two disjoint languages. Say that language $C$ separates $A$ and $B$ if $A \subseteq C$
   and $B \subseteq \overline{C}$. Show that if $A$ and $B$ are disjoint, and $\overline{A}$ and $\overline{B}$ are both Turing-recognizable,
   then some decidable language separates $A$ and $B$.

6. Let $C$ be a language. Prove that $C$ is Turing-recognizable iff a decidable language $D$ exists
   such that $C = \{x | \exists y \in \{0,1\}^* (\langle x, y \rangle \in D)\}$.

7. (optional) Show that every infinite T-recognizable language has an infinite decidable subset.