Read all of Chapters 1 and 2 except Section 2.4.

0.1 Read and solve, but do not turn in: Book, 1.14. [swapping NFA accept/non-accept states]
0.2 Read and solve, but do not turn in: Book, 1.31. [closure under reversal]
0.3 Read and solve, but do not turn in: Book, 1.46b. [Pumping lemma]

You can assume the results from the above problems when solving the problems below.

1. (a) Let $B = \{1^k y \mid y \in \{0, 1\}^* \text{ and } y \text{ contains at least } k \text{ 1s, for } k \geq 1\}$.
   Show that $B$ is a regular language.
   
   (b) Let $C = \{1^k y \mid y \in \{0, 1\}^* \text{ and } y \text{ contains at most } k \text{ 1s, for } k \geq 1\}$.
   Show that $C$ isn’t a regular language.

2. The Hamming distance $H(x, y)$ between two strings $x$ and $y$ of equal length, is the number of corresponding symbols at which $x$ and $y$ differ. For example, $H(1101111, 0001111) = 2$.
   For any language $A$, let $N_1(A) = \{w \mid H(w, x) \leq 1 \text{ for some } x \in A\}$.
   Show that the class of regular languages is closed under the $N_1$ operation.

3. Let $D = \{w \mid w \in \{0, 1\}^* \text{ is not a palindrome (i.e., } w \neq w^R)\}$. Prove that $D$ is not regular.

4. Let $M_1$ and $M_2$ be DFAs that have $k_1$ and $k_2$ states, respectively, and let $U = L(M_1) \cup L(M_2)$.
   
   (a) Show that if $U \neq \emptyset$, then $U$ contains some string $s$, where $|s| < \max(k_1, k_2)$.
   
   (b) Show that if $U \neq \Sigma^*$, then $U$ excludes some string $s$, where $|s| < k_1k_2$.

5. Let $x$ and $y$ be strings over some alphabet $\Sigma$. Say $x$ is a substring of $y$ if $y \in \Sigma^*x\Sigma^*$ and say $x$ is a major substring of $y$ if $x$ is a substring of $y$ and $|x| \geq \frac{1}{2}|y|$.
   For any language $B$, let $MS(B) = \{x \mid x \text{ is a major substring of } y \text{ for some } y \in B\}$.
   Show that if $B$ is regular then $MS(B)$ is context-free.

6. Consider the following CFG $G$:
   $$S \rightarrow aSb \mid aSbb \mid \varepsilon$$
   Describe $L(G)$ and show that $G$ is ambiguous.
   Give an unambiguous grammar $H$ where $L(H) = L(G)$ and prove that $H$ is unambiguous.

7* (optional) Strengthen Problem 5 by showing that if $B$ is regular then $MS(B)$ is also regular.