Problem Set 1

Please turn in each problem on a separate page with your name.

Read all of Chapters 1 and 2 except Section 2.4.

0.1 Read and solve, but do not turn in: Book, 1.14. [swapping NFA accept/non-accept states]

0.2 Read and solve, but do not turn in: Book, 1.31. [closure under reversal]

1. Let \( \Sigma_2 = \{ [0], [1], [1] \} \). Here, \( \Sigma_2 \) contains all columns of 0s and 1s of height two. A string of symbols in \( \Sigma_2 \) gives two rows of 0s and 1s. Consider each row to be a binary number and let

\[
D = \{ w \in \Sigma_2^n | \text{the top row of } w \text{ is a larger number than is the bottom row} \}.
\]

For example, \([0][1][0][1] \in D\), but \([0][1][1][0] \notin D\). Show that \( D \) is regular.

2. Let \( \Sigma_2 \) be the same as in Problem 1. Let

\[
E = \{ w \in \Sigma_2^n | \text{the bottom row of } w \text{ is the reverse of the top row of } w \}.
\]

For example, \([1][0][1][0] \in E\), but \([1][0][0][1] \notin E\). Show that \( E \) is not regular.

3. Let \( x \) and \( y \) be strings over some alphabet \( \Sigma \). Say \( x \) is a substring of \( y \) if \( y \in \Sigma^*x\Sigma^* \).

Define the avoids operation for languages \( A \) and \( B \) to be

\[
A \text{ avoids } B = \{ w | w \in A \text{ and } w \text{ doesn’t contain any string in } B \text{ as a substring} \}.
\]

Prove that the class of regular languages is closed under the avoids operation.

(Hint: Use the closure properties you already know.)

4. Let \( \Sigma = \{0, 1\} \).

(a) Let \( A = \{0^k u 0^k | k \geq 1, u \in \Sigma^* \} \). Show \( A \) is regular.

(b) Let \( B = \{0^k 1 u 0^k | k \geq 1, u \in \Sigma^* \} \). Show \( B \) is not regular.

5. If \( A \) is any language, let \( A_{\frac{1}{2}} \) be the language of all first halves of strings in \( A \) so that

\[
A_{\frac{1}{2}} = \{ x | \text{for some } y, |x| = |y| \text{ and } xy \in A \}.
\]

Show that if \( A \) is regular, then so is \( A_{\frac{1}{2}} \).

6. Let \( \Sigma = \{0, 1\} \) and let \( C_1 = \{ \text{tut} | t \in \Sigma^* \text{ and } u \in 0^*10^* \} \), where \( |t| = |u| \).

Show that \( C_1 \) is a CFL, by giving a CFG and by giving a PDA. You do not need to prove that your solutions work, but please give comments to assist the grader.

(Hint: This problem is somewhat tricky but not complicated. It has a CFG with three rules.)

7.* (\( *= \) optional) If \( A \) is any language, let \( A_{\frac{2}{3}} \) be the set of all strings in \( A \) with their middle thirds removed. Formally \( A_{\frac{2}{3}} = \{ xz \} \) for some \( y, |x| = |y| = |z| \text{ and } xyz \in A \).

Show that if \( A \) is regular, then \( A_{\frac{2}{3}} \) is not necessarily regular.