Read all of Chapters 1 and 2 except Section 2.4.

0.1 Read and solve, but do not turn in: Book, 1.14. [swapping NFA accept/non-accept states]

0.2 Read and solve, but do not turn in: Book, 1.31. [closure under reversal]

0.3 Read and solve, but do not turn in: Book, 1.46c. [Pumping lemma]

1. Let $\Sigma_2 = \{ \begin{bmatrix} 0 \end{bmatrix}, \begin{bmatrix} 0 \end{bmatrix}, \begin{bmatrix} 1 \end{bmatrix}, \begin{bmatrix} 1 \end{bmatrix} \}$. Here, $\Sigma_2$ contains all columns of 0s and 1s of height two. A string of symbols in $\Sigma_2$ gives two rows of 0s and 1s. Consider each row to be a binary number and let

   $$ D = \{ w \in \Sigma_2^* \mid \text{the top row of } w \text{ is a larger number than is the bottom row} \}. $$

   For example, $\begin{bmatrix} 0 \end{bmatrix} \begin{bmatrix} 0 \end{bmatrix} \begin{bmatrix} 1 \end{bmatrix} \begin{bmatrix} 0 \end{bmatrix} \in D$, but $\begin{bmatrix} 0 \end{bmatrix} \begin{bmatrix} 0 \end{bmatrix} \begin{bmatrix} 1 \end{bmatrix} \begin{bmatrix} 0 \end{bmatrix} \notin D$. Show that $D$ is regular.

2. Let $\Sigma_2$ be the same as in Problem 1. Let

   $$ E = \{ w \in \Sigma_2^* \mid \text{the bottom row of } w \text{ is the reverse of the top row of } w \}. $$

   For example, $\begin{bmatrix} 1 \end{bmatrix} \begin{bmatrix} 1 \end{bmatrix} \begin{bmatrix} 0 \end{bmatrix} \begin{bmatrix} 1 \end{bmatrix} \in E$, but $\begin{bmatrix} 1 \end{bmatrix} \begin{bmatrix} 1 \end{bmatrix} \begin{bmatrix} 0 \end{bmatrix} \begin{bmatrix} 0 \end{bmatrix} \notin E$. Show that $E$ is not regular.

3. Let $A$ be any language. Define $\text{DROP-ONE}(A)$ to be the language containing all strings that can be obtained by removing one symbol from a string in $A$.

   Thus, $\text{DROP-ONE}(A) = \{ xz \mid xyz \in A \text{ where } x, z \in \Sigma^*, y \in \Sigma \}$.

   Show that the class of regular languages is closed under the $\text{DROP-ONE}$ operation.

   Give both a proof by picture and a more formal proof by construction as in Theorem 1.47.

4. Let $\Sigma = \{0, 1\}$. Let $WW_k = \{ w w \mid w \in \Sigma^* \text{ and } w \text{ is of length } k \}$.

   (a) Show that for each $k$, no DFA can recognize $WW_k$ with fewer than $2^k$ states.

   (b) Describe a much smaller NFA for $\overline{WW_k}$, the complement of $WW_k$.

5. Let $\Sigma = \{0, 1\}$.

   (a) Let $A = \{ 0^k u 0^k \mid k \geq 1 \text{ and } u \in \Sigma^* \}$. Show $A$ is regular.

   (b) Let $B = \{ 0^k 1 u 0^k \mid k \geq 1 \text{ and } u \in \Sigma^* \}$. Show $B$ is not regular.

6. Let $\Sigma = \{0, 1\}$ and let $C_1 = \{ t r t \mid t \in 0^* \text{ and } r \in 0^* 10^* \}, \text{where } |t| = |r|$. The notation $|x|$ means the length of string $x$. Show that $C_1$ is a CFL, by giving a CFG and by giving a PDA. You do not need to prove that your solutions work, but please give comments to assist the grader. (Hint: This problem is tricky but not complicated. It has a CFG with three rules.)

7. $(\ast = \text{optional})$ For any language $A$ let $RC(A) = \{ y x \mid xy \in A \}$.

   Show that the class of CFLs is closed under the $RC$ operation.