

Problem Set 1

Please turn in each problem on a separate page with your name.

- 0.1 Read and solve, but do not turn in: Book, 1.14 . [swapping NFA accept/non-accept states]
- 0.2 Read and solve, but do not turn in: Book, 2.18 . [CFL \cap regular = CFL]
You can check your solution with the one in the book.
- 0.3 Read and solve, but do not turn in: Book, 2.26 . [Chomsky normal form]
1. The *Hamming distance* $H(x, y)$ between two strings x and y of equal length is the number of corresponding symbols at which the strings differ. For example, $H(1101111, 0001111) = 2$. For any language A , let $N_1(A) = \{w \mid H(w, x) \leq 1 \text{ for some } x \in A\}$. Show that the class of regular languages is closed under the N_1 operation.
 2. Let $M10 = \{a^{10k} \mid k \geq 0\}$ and let $\overline{M10} = \{a^i \mid i \text{ is not a multiple of } 10\}$. Here $\Sigma = \{a\}$, a^l means $aaa \cdots a$ of length l , and $\overline{M10}$ is the complement of $M10$.
 - (a) Give the state diagram of an 8-state NFA which recognizes $\overline{M10}$.
 - (b) Prove that any NFA which recognizes $M10$ has at least 10 states.
 3. Let $Z = \{w \mid w = xy \text{ for } |x| = |y| \text{ and } x \text{ and } y \text{ contain equal numbers of } 1\text{s}\}$. Show that Z is not a regular language. Assume $\Sigma = \{0, 1\}$.
 4. A *Forward-Backward Deterministic Finite Automaton* (FB-DFA) has two read heads, one reading the input forward and the other simultaneously reading the input backward. The FB-DFA moves from state to state, depending on the symbols being read by both heads. The machine accepts its input if it is in an accept state when the forward head reaches the end of the input (coincides with the backward head reaching the beginning of the input).
 - (a) Give a formal definition of the FB-DFA model as a 5-tuple, including the form of the transition function, and a formal description of its computation and accepted language.
 - (b) Extending our notion of state diagram to FB-DFA, give the state diagram for a machine recognizing $B = \{0^k 1^k \mid k \geq 0\}$.
 5. Using the procedures we have given in class and in the book, give rough but reasonable estimates in the following problems. Explain your reasoning. (For example, your answers might take the form $O(2^n)$, or $O(n^5)$, or $O(n)$. If you aren't familiar with O -notation, look at Page 248 in the text. You can ignore low-order effects in your solution.)
 - (a) When converting a regular expression with n symbols to an NFA, how many states does the resulting NFA have?
 - (b) When converting a DFA with n states to a regular expression, how many symbols does the resulting expression have?
 6. Book, 2.19 . [tricky grammar]
 7. Book, 2.27 . [if-then-else ambiguous grammar]
You do not need to prove your grammar works, but please add a few comments about why it does work to help the grader.
 - 8* (star means optional) Read Problem 4. Prove that the language of any FB-DFA is a CFL.