RANDOM PLANAR MATCHING AND BIN PACKING

by

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ABSTRACT

This thesis solves several well-known random planar matching problems, and then applies the results to determine the average case behavior of two of the most commonly used on-line algorithms for bin packing. In the planar matching problems two kinds of points are distributed randomly in a unit square. We investigate the length of the edges in an optimal matching between the two kinds of points. Planar matching problems have arisen in several diverse areas of mathematics, including probability, VLSI layout, and analysis of algorithms. We define four planar matching problems: average edge length matching, rightward matching, up-right matching and maximum edge length matching. These problems differ in the constraints we impose on the matchings and in the measure with respect to which the matching is optimized. We find new tight bounds for up-right and maximum edge length matching.

We then use these matching problem to analyze the behavior of several on-line bin packing algorithms. We find tight bounds on the expected wasted space in the Best Fit algorithm, and bounds differing by a small factor for the expected wasted space in the First Fit algorithm. We also provide a new lower bound for the performance on any on-line algorithm. The performance of Best Fit is very close to this theoretical lower bound.

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