18.440 PROBLEM SET 9: DUE MAY 2

A. FROM TEXTBOOK CHAPTER EIGHT:

1. Problem 7: A person has 100 light bulbs whose lifetimes are independent exponentials with mean 5 hours. If the bulbs are used one at a time, with a failed bulb being replaced immediately by a new one, approximate the probability that there is still a working bulb after 525 hours.

2. Problem 15: An insurance company has 10,000 automobile policyholders. The expected yearly claim per policy-holder is $240, with a standard deviation of $800. Approximate the probability that the total yearly claim exceeds $2.7 million.

3. Theoretical Exercise 8: Explain why a gamma random variable with parameters (t, λ) has an approximately normal distribution when t is large.

4. Problem/Theoretical Exercises 9: Suppose a fair coin is tossed 1000 times. If the first 100 tosses all result in heads, what proportion of heads would you expect on the final 900 tosses? Comment on the statement "The strong law of large numbers swamps, but does not compensate."

5. Problem/Theoretical Exercises 13: Show that if $E[X] < 0$ and $\theta \neq 0$ is such that $E[e^{\theta X}] = 1$, then $\theta > 0$.

B. FROM TEXTBOOK CHAPTER NINE:

1. Problem/Theoretical Exercises 7: A transition matrix is said to be doubly stochastic if $\sum_{i=0}^{M} P_{ij} = 1$ for all states $j = 0, 1, \ldots, M$. Show that if such a Markov chain is ergodic, then $\prod_j = 1/(M+1), j = 0, 1, \ldots, M$.

2. Problem/Theoretical Exercises 9: Suppose that whether it rains tomorrow depends on past weather conditions only through the last 2 days. Specifically, suppose that if it has rained yesterday and today, then it will rain tomorrow with probability .8; if it rained yesterday but not today, then it will rain tomorrow with probability .3; if it rained today but not yesterday, then it will rain tomorrow with probability .4; and if it has not rained either yesterday or today, then it will rain tomorrow with probability .2. Over the long term, what proportion of days does it rain?