18.440 PROBLEM SET FIVE, DUE MARCH 21

A. FROM TEXTBOOK CHAPTER FOUR:

1. Problem 70: At time 0 a coin that comes up heads with probability p is flipped and falls to the ground. Suppose it lands on heads. At times chosen according to a Poisson process with rate λ, the coin is picked up and flipped. (Between these times the coin remains on the ground.) What is the probability that the coin is on its head side at time t? *Hint:* What would be the conditional probability if there were no additional flips by time t, and what would it be if there were additional flips by time t?

2. Problem 84: Suppose that 10 balls are put into 5 boxes, with each ball independently being put in box i with probability p_i, $\sum_{i=1}^{5} p_i = 1$.
   (a) Find the expected number of boxes that do not have any balls.
   (b) Find the expected number of boxes that have exactly 1 ball.

3. Theoretical Exercise 16: Let X be a Poisson random variable with parameter λ. Show that $P\{X = i\}$ increases monotonically and then decreases monotonically as i increases, reaching its maximum when i is the largest integer not exceeding λ. *Hint:* Consider $P\{X = i\}/P\{X = i - 1\}$.

4. Theoretical Exercise 25: Suppose that the number of events that occur in a specified time is a Poisson random variable with parameter λ. If each event is "counted" with probability p, independently of every other event, show that the number of events that are counted is a Poisson random variable with parameter λp. Also, give an intuitive argument as to why this should be so. As an application of the preceding result, suppose taht the number of distinct uranium deposits in a given area is a Poisson random variable with parameter $\lambda = 10$. If, in a fixed period of time, each deposit is discovered independently with probability $\frac{1}{50}$, find the probability that (a) exactly 1, (b) at least 1, and (c) at most 1 deposit is discovered during that time.

B. FROM TEXTBOOK CHAPTER FIVE:

1. Problem 8: The lifetime in hours of an electronic tube is a random variable having a probability density function given by

$$f(x) = xe^{-x} \quad x \geq 0$$

   Compute the expected lifetime of such a tube.

2. Problem 11: A point is chosen at random on a line segment of length L. Interpret this statement, and find the probability that the ratio of the shorter to the longer segment is less than $1/4$.

C. ANSWER THE FOLLOWING:

1. Compute the expectation of X^n where n is a positive integer and X is a uniform random variable on the interval $[0, 1]$.

2. How does the answer change if the random variable is instead taken to be uniform on $[0, L]$ for some constant L?