

**Mathematics of Finance**  
**Problem Set 7**  
**Due November 16**

I. Complete Exercises 8.5, 8.6, 8.10, and 8.11 of Zawstaniak and Capiński.

II. In the binomial tree model, assume  $A(0) = S(0) = 1$ , and  $r = .05$ ,  $u = .1$  and  $d = -.05$ . Let  $C(n)$  represent the price at time  $n$  of a call option with exercise time 5 and strike price 1.

- (a) Compute the risk neutral probability  $p_*$  that the stock goes up at a given stage.
- (b) On a full sheet of paper, draw a big tree showing the possible values of  $S(n)/A(n)$  for each  $0 \leq n \leq 5$ .
- (c) On another full sheet of paper, draw the tree showing the values of  $C(n)/A(n)$  for  $0 \leq n \leq 5$ . Do this by first computing the values for  $n = 5$  and then using the risk neutral probability and martingale property to fill in the values corresponding to the other nodes in the tree.
- (d) What is  $C(0)$ ?

III. Let  $f(x) = |10 - x| + |20 - x| + |30 - x|$ . Sketch a graph of  $f(x)$ . Let  $S(T)$  denote the value of a stock at time  $T$ . Describe a portfolio made up positive quantities of European calls and puts (with exercise time  $T$  and strike prices that you specify) such that the value of the portfolio at time  $T$  is  $f(S(T))$ . Is it possible to create such a portfolio for *every* convex, positive, piecewise linear function  $f$ ?

IV. Look up the options prices for Google stock, e.g., by going to [www.cboe.com](http://www.cboe.com), clicking "delayed quotes" and typing in the stock symbol GOOG.

- (a) Look at the calls that expire in January of 2007 and graph the function  $f$  that gives the current price of a January, 2007 call (when they differ, use the average of the bid and the ask price) as a function of its strike price.
- (b) Sketch the first and second derivatives of  $f$ . (This will be an approximation, since you only know the value of  $f$  at a discrete set of

points.) What do these tell you about the risk neutral probability distribution of the price of Google at the exercise date in January, 2007? (You may assume for simplicity that the risk-free interest rate is constant at 4.5 percent and that no dividends are paid.)

- (c) Using the data above, estimate the risk neutral probability that Google stock will be worth more than five hundred dollars per share by January 2007.
- (d) A friend at a party says, “Dude, Google stock is so unstoppable. I use Google all the time. It’s got to have at least a one in three chance of being above 500 on the option exercise date in January of 2007. If you give me two to one odds — i.e., you give me  $2X$  if I’m right and I give you  $X$  if I’m wrong — then I’ll bet you any amount  $X > 0$  that you want that Google is over 500 at that time.” Explain how you can make a bet with your friend and also buy or sell short either Google stock, Google call options, or risk free assets in such a way that guarantees you a risk free profit with zero initial investment. Take into account that you can only buy at the ask price and sell at the bid price and that the options are American options. (Hint: first argue that the fact that they are American options doesn’t really matter.)
- (e) Another friend says, “No way is Google going to do that well. It’s got at most a ten percent chance of being above 500 on the exercise date in January, 2007. I’ll bet you any amount you want that it *doesn’t* beat 500, and I’ll give you ten to one odds.” Explain how you can take the bet with your friend and also buy or sell short Google stock, Google call options, or risk free assets in such a way that guarantees you a risk free profit with zero initial investment.