Mathematics of Finance
Problem Set 4
Due October 26

I. Complete Exercises 5.3, 5.5, 5.9, 5.10, 5.11, 5.12, 5.14, 5.15 of Zawstaniak and Capiński.

II. Suppose that there are one hundred stocks. Let K_i, for $1 \leq i \leq 100$, denote their returns over a fixed period. Suppose that $\mu_i = \mathbb{E}K_i = .05$ and $\sigma_i = \sqrt{\mathrm{Var}K_i} = .1$ for all $1 \leq i \leq 100$. Suppose further that $\rho_{i,j} = \frac{Cov(K_i,K_j)}{\sigma_i \sigma_j} = a$ for all $i \neq j$.

(a) Find the minimum variance portfolio and compute the mean μ_V and standard deviation of its return, K_V, as functions of a.

(b) If you have done this correctly, then when $a = 1$, the standard deviation of K_V should be equal to .1 (same as for a single stock) and when $a = 0$ it should be equal to .01 (one tenth of the value for of a single stock). Explain why. What happens when a is negative (but not so negative that the matrix fails to be positive definite)?

III. Use the same setup as the previous problem but suppose now that $\sigma_i^2 = \mu_i = i$ for all $1 \leq i \leq 100$ but $\rho_{i,j} = 0$ for all $i \neq j$.

1. Compute the minimum variance portfolio in this case.

2. Compute the highest expected return portfolio in this case, assuming that short selling is not allowed.

3. Generalize the answer in (a) to general values of σ_i^2 (but still assuming that $\rho_{i,j} = 0$ for all $i \neq j$).