

Limit Theorems
Problem Set 7
Due November 15

1. Prove that for each $\alpha \in (0, 2]$ there is an infinitely divisible probability distribution μ_α on \mathbb{R} with characteristic function $\phi(t) = e^{-|t|^\alpha}$. These are called the Lévy symmetric α -stable distributions.
2. Give the Lévy Khintchine representation for general α .
3. Show that the Cauchy distribution is obtained when $\alpha = 1$ and the Gaussian distribution when $\alpha = 2$.
4. Prove that the Lévy symmetric α -stable distributions (and constant rescalings) are the only symmetric-about-zero probability distributions μ on \mathbb{R} with the property that for all n the convolution of n copies of μ is a constant rescaling of μ .
5. Prove that the variance is finite only when $\alpha = 2$.
6. Find out more about these distributions and their generalizations by reading, e.g., Wikipedia's article on Levy skew stable distributions.