Limit Theorems
Problem Set 1
Due September 20

1. Complete steps 2, 3, and 4 of the proof of the Caratheodory Extension Theorem given in Section 1.2 of Professor Varadhan's lecture notes, along with Exercise 1.8 (which ends the proof).

2. Let X be a bounded open subset of $\mathbb{R}^d$ and P Lebesgue measure on X (divided by the volume of X, so that $P(X) = 1$). Suppose that the functions $f_n : X \to \mathbb{R}$ are **uniformly Lipschitz**, i.e., they satisfy $|f_n(x) - f_n(y)| \leq C||x - y||$ for a constant $C > 0$ that is independent of n, x, and y. ***Suppose further that*** $f : X \to \mathbb{R}$ ***is continuous.*** Prove that in this case, $f_n \to f$ pointwise if and only if $f_n \to f$ in probability.