18.440 PROBLEM SET 8: DUE NOVEMBER 23

A. FROM TEXTBOOK CHAPTER SEVEN:

1. Problems: 50, 61, 65, 75, 78
2. Theoretical Exercises: 27, 28, 46, 49, 55

B. (Just for fun — not to hand in) Let $V = (V_1, V_2, \dots, V_n)$ be a random vector whose components V_i are independent, identically distributed normal random variables of mean zero, variance one. Note that the density function for V may be written as $f(v) = (2\pi)^{-n/2}e^{-|v|^2/2}$ where $v = (v_1, v_2, \dots, v_n)$ and $|v|^2 = v_1^2 + v_2^2 + \dots + v_n^2$.

1. Let M be an $n \times n$ matrix. Write $W = MV$ and compute the mean and covariance of W_i for each $1 \le i \le n$.
2. Write the probability density function for W.
3. Classify the set of matrices M for which MV has the same probability density function as V.
4. Is every n-dimensional mean zero multivariate normal distribution (as defined in Section 7.8 of the textbook) the distribution of MV for some choice of M? If so, to what extent does the distribution uniquely determine M?

C.(Just for fun — not to hand in) Try to formulate and prove a version of the central limit theorem that shows that sums of independent heavy-tailed random variables (divided by appropriate constants) converge in law to a stable random variable (instead of a normal random variable). See for example `http://eom.springer.de/A/a013920.htm` or the wikipedia articles on stable distributions for definitions and hints. You will need to use characteristic functions instead of the moment generating function.