18.440 PROBLEM SET SIX, DUE NOVEMBER 4

A. FROM TEXTBOOK CHAPTER FIVE:

1. Problems: 6, 20, 23, 32

2. Theoretical Exercises: 9, 21, 29, 30, 31

B. At time zero, a single bacterium in a dish divides into two bacteria. This species of bacteria has the following property: after a bacterium B divides into two new bacteria B_1 and B_2, the subsequent length of time until each B_i divides is an exponential random variable of rate $\lambda = 1$, independently of everything else happening in the dish.

1. Compute the expectation of the time T_n at which the number of bacteria reaches n.

2. Compute the variance of T_n.

3. Are both of the answers above unbounded, as functions of n? Give a rough numerical estimate of the values when $n = 10^{50}$.

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