18.600: Lecture 1

Permutations and combinations, Pascal's triangle, learning to count

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Outline

Remark, just for fun

Permutations

Counting tricks

Binomial coefficients

Problems

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Politics

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- Suppose $X(t)$ is known to vary continuously in time. What is the probability it will reach 59 before reaching 57?
- "Efficient market hypothesis" suggests about .5.
- Reasonable model: use sequence of fair coin tosses to decide the order in which $X(t)$ passes through different integers.

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- Let's start with easier questions.

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- $n \cdot (n-1) \cdot (n-2) \ldots (n-k+1) = n!/(n-k)!$
- A **permutation** is a map from $\{1, 2, \ldots, n\}$ to $\{1, 2, \ldots, n\}$. There are $n!$ permutations of n elements.

Permutation notation

- A **permutation** is a function from $\{1, 2, \ldots, n\}$ to $\{1, 2, \ldots, n\}$ whose range is the whole set $\{1, 2, \ldots, n\}$. If σ is a permutation then for each j between 1 and n, the the value $\sigma(j)$ is the number that j gets mapped to.

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- If σ and ρ are both permutations, write $\sigma \circ \rho$ for their composition. That is, $\sigma \circ \rho(j) = \sigma(\rho(j))$.

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- A permutation is "fixed point free" if there are no cycles of length one.

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Fundamental counting trick

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- This is a useful trick: break counting problem into a sequence of stages so that one always has the same number of choices to make at each stage. Then the total count becomes a product of number of choices available at each stage.
- Easy to make mistakes. For example, maybe in your problem, the number of choices at one stage actually *does* depend on choices made during earlier stages.

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- If you have 5 indistinguishable black cards, 2 indistinguishable red cards, and three indistinguishable green cards, how many distinct shuffle patterns of the ten cards are there?
- Answer: if the cards were distinguishable, we'd have 10!. But we're overcounting by a factor of 5!2!3!, so the answer is 10!/(5!2!3!).

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- Answer: $\binom{n}{k} := \frac{n!}{k!(n-k)!}$

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- Question: what is $\sum_{k=0}^{n} \binom{n}{k}$?
- Answer: $(1+1)^n = 2^n$.

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- How many 10 digit numbers with no consecutive digits that agree?
- If initial digit can be zero, have $10 \cdot 9^9$ ten-digit sequences. If initial digit required to be non-zero, have 9^{10}.
- How many 10 digit numbers (allowing initial digit to be zero) in which only 5 of the 10 possible digits are represented?
- This is one is tricky, can be solved with *inclusion-exclusion* (to come later in the course).
- How many ways to assign a birthday to each of 23 distinct people? What if no birthday can be repeated?

More problems

- How many hands that have four cards of the same suit, one card of another suit?
- $4\binom{13}{4} \cdot 3\binom{13}{1}$
- How many 10 digit numbers with no consecutive digits that agree?
- If initial digit can be zero, have $10 \cdot 9^9$ ten-digit sequences. If initial digit required to be non-zero, have 9^{10}.
- How many 10 digit numbers (allowing initial digit to be zero) in which only 5 of the 10 possible digits are represented?
- This is one is tricky, can be solved with *inclusion-exclusion* (to come later in the course).
- How many ways to assign a birthday to each of 23 distinct people? What if no birthday can be repeated?
- 366^{23} if repeats allowed. $366!/343!$ if repeats not allowed.