

## 18.440 PROBLEM SET TWO, DUE SEPTEMBER 25

### A. FROM TEXTBOOK CHAPTER TWO:

1. Problems: 29, 35, 48, 51, 54.
2. Theoretical Exercises: 18, 20, 21.
3. Self-Test Problems and Exercises: 11.

B. Toss a fair coin  $n$  times and let  $X_i$  be equal to 1 if the  $i$ th coin toss is heads and  $-1$  if it is tails. Write  $S_k = \sum_{i=1}^k X_i$ . Compute the probability of each of the following events (as a function of  $n$ ).

1.  $S_k$  is non-negative for all  $k \in \{0, 1, 2, \dots, n\}$ .
2.  $S_k < 17$  for all  $k \in \{0, 1, 2, \dots, n\}$ .

If you get stuck on this problem, try googling “random walk reflection principle.”

C. (\*) The following is a popular and rather instructive puzzle. A standard deck of 52 cards (26 red and 26 black) is shuffled so that all orderings are equally likely. We then play the following game: I begin turning the cards over one at a time so that you can see them. At some point (before I have turned over all 52 cards) you say “I’m ready!” At this point I turn over the next card and if the card is red, you receive one dollar; otherwise you receive nothing. You would like to design a strategy to maximize the probability that you will receive the dollar. How should you decide when to say “I’m ready”?