

# 18.440: Lecture 14

## More discrete random variables

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# Outline

Geometric random variables

Negative binomial random variables

Problems

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- ▶ Can you prove directly that these probabilities sum to one?
- ▶ Say  $X$  is a **geometric random variable with parameter  $p$** .

## Geometric random variable expectation

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- ▶ Kind of makes sense.  $X - 1$  is “number of extra tosses after first.” Given first coin heads (probability  $p$ ),  $X - 1$  is 0. Given first coin tails (probability  $q$ ), conditional law of  $X - 1$  is geometric with parameter  $p$ . In latter case, conditional expectation of  $X - 1$  is same as a priori expectation of  $X$ .



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- ▶ Thus  $E[X] - 1 = E[X - 1] = p \cdot 0 + qE[X] = qE[X]$  and solving for  $E[X]$  gives  $E[X] = 1/(1 - q) = 1/p$ .

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- ▶  $\text{Var}[X] = (2-p)/p^2 - 1/p^2 = (1-p)/p^2 = 1/p^2 - 1/p = q/p^2$ .

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- ▶ Takes  $1/p$  coin tosses on average to see a heads.

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- ▶ So  $P\{X = k\} = \binom{k-1}{r-1} p^{r-1} (1-p)^{k-r} p$ . Can you prove these sum to 1?
- ▶ Call  $X$  **negative binomial random variable with parameters  $(r, p)$** .

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- ▶ Write  $X = X_1 + X_2 + \dots + X_r$  where  $X_k$  is number of tosses (following  $(k-1)$ th head) required to get  $k$ th head. Each  $X_k$  is geometric with parameter  $p$ .

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- ▶ How about  $\text{Var}[X]$ ?
- ▶ Turns out that  $\text{Var}[X] = \text{Var}[X_1] + \text{Var}[X_2] + \dots + \text{Var}[X_r]$ .  
So  $\text{Var}[X] = rq/p^2$ .

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- ▶ **Geometric random variables:** What's the probability baby is quiet from midnight to three?



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- ▶ **Negative binomial expectation:** How many minutes do I expect to wait until the fifth cry?
- ▶ **Poisson approximation:** Approximate the probability there are exactly five cries during the night.
- ▶ **Exponential random variable approximation:** Approximate probability baby quiet all night.

## More fun problems

- ▶ Suppose two soccer teams play each other. One team's number of points is Poisson with parameter  $\lambda_1$  and other's is independently Poisson with parameter  $\lambda_2$ . (You can google “soccer” and “Poisson” to see the academic literature on the use of Poisson random variables to model soccer scores.) Using Mathematica (or similar software) compute the probability that the first team wins if  $\lambda_1 = 2$  and  $\lambda_2 = 1$ . What if  $\lambda_1 = 2$  and  $\lambda_2 = .5$ ?

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- ▶ Imagine you start with the number 60. Then you toss a fair coin to decide whether to add 5 to your number or subtract 5 from it. Repeat this process with independent coin tosses until the number reaches 100 or 0. What is the *expected* number of tosses needed until this occurs?