18.440: Lecture 1

Permutations and combinations, Pascal's triangle, learning to count

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Outline

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Remark, just for fun

Permutations

Counting tricks

Binomial coefficients

Problems

Politics

- Suppose that betting markets place the probability that your favorite presidential candidates will be elected at 58 percent. Price of a contact that pays 100 dollars if your candidate wins is 58 dollars.

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- "Efficient market hypothesis" suggests about .5.
- Reasonable model: use sequence of fair coin tosses to decide the order in which $X(t)$ passes through different integers.

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- All of the math in this course has a lot of applications.

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- $n \cdot (n-1) \cdot (n-2) \ldots (n-k+1) = n!/(n-k)!$
- A **permutation** is a map from $\{1, 2, \ldots, n\}$ to $\{1, 2, \ldots, n\}$. There are $n!$ permutations of n elements.

Permutation notation

- A **permutation** is a function from $\{1, 2, \ldots, n\}$ to $\{1, 2, \ldots, n\}$ whose range is the whole set $\{1, 2, \ldots, n\}$. If σ is a permutation then for each j between 1 and n, the the value $\sigma(j)$ is the number that j gets mapped to.

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- If σ and ρ are both permutations, write $\sigma \circ \rho$ for their composition. That is, $\sigma \circ \rho(j) = \sigma(\rho(j))$.

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- If you pick some j and repeatedly apply σ to it, it will "cycle through" the numbers in its cycle.
- Generally, a function is called an **involution** if $f(f(x)) = x$ for all x.
- A permutation is an involution if all cycles have length one or two.
- A permutation is "fixed point free" if there are no cycles of length one.

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Fundamental counting trick

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- This is a useful trick: break counting problem into a sequence of stages so that one always has the same number of choices to make at each stage. Then the total count becomes a product of number of choices available at each stage.
- Easy to make mistakes. For example, maybe in your problem, the number of choices at one stage actually *does* depend on choices made during earlier stages.

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- If you have 5 indistinguishable black cards, 2 indistinguishable red cards, and three indistinguishable green cards, how many distinct shuffle patterns of the ten cards are there?
- Answer: if the cards were distinguishable, we'd have $10!$. But we're overcounting by a factor of $5!2!3!$, so the answer is $10!/(5!2!3!)$.

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- Answer: $\binom{n}{k}$.
- $(x+1)^n = \binom{n}{0} \cdot 1 + \binom{n}{1}x^1 + \binom{n}{2}x^2 + \ldots + \binom{n}{n-1}x^{n-1} + \binom{n}{n}x^n$.

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- Question: what is $\sum_{k=0}^{n} \binom{n}{k}$?
- Answer: $(1+1)^n = 2^n$.

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- 366^{23} if repeats allowed. $366!/343!$ if repeats not allowed.