Eight athletic teams are ranked 1 through 8 after season one, and ranked 1 through 8 again after season two. Assume that each set of rankings is chosen uniformly from the set of 8! possible rankings and that the two rankings are independent. Let $N$ be the number of teams whose rank does not change from season one to season two. Let $N_+$ the number of teams whose rank improves by exactly two spots. Let $N_-$ be the number whose rank declines by exactly two spots. Compute the following:

$E[N]$, $E[N_+]$, and $E[N_-]$

$\text{Var}[N]$, $\text{Var}[N_+]$
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- $E[N]$, $E[N_+]$, and $E[N_-]$
- $\text{Var}[N]$
- $\text{Var}[N_+]$
Let $N_i$ be 1 if team ranked $i$th first season remains $i$th second seasons. Then $E[N] = E[\sum_{i=1}^{8} N_i] = 8 \cdot \frac{1}{8} = 1$. Similarly, $E[N+] = E[N-] = 6 \cdot \frac{1}{8} = 3/4$.
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$\text{Var}[N] = E[N^2] - E[N]^2$ and $E[N^2] = E[\sum_{i=1}^{8} \sum_{j=1}^{8} N_i N_j] = 8 \cdot \frac{1}{8} + 56 \cdot \frac{1}{56} = 2$. 
Expectation and variance — answers

- Let $N_i$ be 1 if team ranked $i$th first season remains $i$th second seasons. Then $E[N] = E[\sum_{i=1}^{8} N_i] = 8 \cdot \frac{1}{8} = 1$. Similarly, $E[N_+] = E[N_-] = 6 \cdot \frac{1}{8} = 3/4$.

- $\text{Var}[N] = E[N^2] - E[N]^2$ and $E[N^2] = E[\sum_{i=1}^{8} \sum_{j=1}^{8} N_iN_j] = 8 \cdot \frac{1}{8} + 56 \cdot \frac{1}{56} = 2$.

- $N^i_+$ be 1 if team ranked $i$th has rank improve to $(i - 2)$th for second seasons. Then $E[(N_+)^2] = E[\sum_{j=1}^{8} \sum_{3=1}^{8} N^i_+ N^j_+] = 6 \cdot \frac{1}{8} + 30 \cdot \frac{1}{56} = 9/7$, so $\text{Var}[N_+] = 9/7 - (3/4)^2$. 

Roll ten dice. Find the conditional probability that there are exactly 4 ones, given that there are exactly 4 sixes.
Straightforward approach: $P(A|B) = P(AB)/P(B)$. 
Straightforward approach: \( P(A|B) = \frac{P(AB)}{P(B)} \).

Numerator: is \( \binom{10}{4}\binom{6}{4}^2 \). Denominator is \( \binom{10}{4}\binom{5}{6}^2 \).
Straightforward approach: \( P(A|B) = \frac{P(AB)}{P(B)} \).

Numerator: is \( \left( \begin{array}{c} 10 \\ 4 \end{array} \right) \left( \begin{array}{c} 6 \\ 4 \end{array} \right) 4^2 \). Denominator is \( \left( \begin{array}{c} 10 \\ 4 \end{array} \right) 5^6 \).

Ratio is \( \left( \begin{array}{c} 6 \\ 4 \end{array} \right) 4^2 / 5^6 = \left( \begin{array}{c} 6 \\ 4 \end{array} \right) (\frac{1}{5})^4 (\frac{4}{5})^2 \).
Conditional distributions — answers

- Straightforward approach: \( P(A|B) = \frac{P(AB)}{P(B)} \).
- Numerator: is \( \frac{\binom{10}{4}\binom{6}{4}^2}{6^{10}} \). Denominator is \( \frac{\binom{10}{4}5^6}{6^{10}} \).
- Ratio is \( \binom{6}{4}^2/5^6 = \binom{6}{4} \left(\frac{1}{5}\right)^4 \left(\frac{4}{5}\right)^2 \).
- Alternate solution: first condition on location of the 6’s and then use binomial theorem.
Suppose that in a certain town earthquakes are a Poisson point process, with an average of one per decade, and volcano eruptions are an independent Poisson point process, with an average of two per decade. The \( V \) be length of time (in decades) until the first volcano eruption and \( E \) the length of time (in decades) until the first earthquake. Compute the following:

- \( \mathbb{E}[E^2] \) and \( \text{Cov}[E, V] \).
Suppose that in a certain town earthquakes are a Poisson point process, with an average of one per decade, and volcano eruptions are an independent Poisson point process, with an average of two per decade. The $V$ be length of time (in decades) until the first volcano eruption and $E$ the length of time (in decades) until the first earthquake. Compute the following:

- $E[E^2]$ and $\text{Cov}[E, V]$.
- The expected number of calendar years, in the next decade (ten calendar years), that have no earthquakes and no volcano eruptions.
Suppose that in a certain town earthquakes are a Poisson point process, with an average of one per decade, and volcano eruptions are an independent Poisson point process, with an average of two per decade. The $V$ be length of time (in decades) until the first volcano eruption and $E$ the length of time (in decades) until the first earthquake. Compute the following:

- $\mathbb{E}[E^2]$ and $\text{Cov}[E, V]$.
- The expected number of calendar years, in the next decade (ten calendar years), that have no earthquakes and no volcano eruptions.
- The probability density function of $\min\{E, V\}$. 

Poisson point processes
$E[E^2] = 2$ and $\text{Cov}[E, V] = 0$. 

Probability of no earthquake or eruption in first year is $e^{-2+1} = e^{-\frac{3}{10}}$. Same for any year by memoryless property. Expected number of quake/eruption-free years is $10e^{-\frac{3}{10}} \approx 7.4$. 

Probability density function of min $\{E, V\}$ is $3e^{-2+1}x$ for $x \geq 0$, and 0 for $x < 0$. 
- $E[E^2] = 2$ and $\text{Cov}[E, V] = 0$.
- Probability of no earthquake or eruption in first year is 
  $e^{-(2+1)\frac{1}{10}} = e^{-0.3}$ (see next part). Same for any year by memoryless property. Expected number of 
  quake/eruption-free years is $10e^{-0.3} \approx 7.4$. 
\( E[E^2] = 2 \) and \( \text{Cov}[E, V] = 0 \).

Probability of no earthquake or eruption in first year is
\[ e^{-(2+1)\frac{1}{10}} = e^{-\cdot 3} \] (see next part). Same for any year by memoryless property. Expected number of quake/eruption-free years is \( 10e^{-\cdot 3} \approx 7.4 \).

Probability density function of min\( \{E, V\} \) is \( 3e^{-(2+1)x} \) for \( x \geq 0 \), and 0 for \( x < 0 \).
Order statistics

Let $X$ be a uniformly distributed random variable on $[-1, 1]$. 
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Compute the variance of $X^2$. 
Order statistics

Let $X$ be a uniformly distributed random variable on $[-1, 1]$.

- Compute the variance of $X^2$.
- If $X_1, \ldots, X_n$ are independent copies of $X$, what is the probability density function for the smallest of the $X_i$?
Order statistics — answers

\[ \text{Var}[X^2] = E[X^4] - (E[X^2])^2 \]
\[ = \int_{-1}^{1} \frac{1}{2} x^4 \, dx - \left( \int_{-1}^{1} \frac{1}{2} x^2 \, dx \right)^2 = \frac{1}{5} - \frac{1}{9} = \frac{4}{45}. \]
Order statistics — answers

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Note that for \( x \in [-1, 1] \) we have
\[ P\{X > x\} = \int_{x}^{1} \frac{1}{2} \, dx = \frac{1 - x}{2}. \]

If \( x \in [-1, 1] \), then
\[ P\{\min\{X_1, \ldots, X_n\} > x\} = P\{X_1 > x, X_2 > x, \ldots, X_n > x\} = \left( \frac{1 - x}{2} \right)^n. \]
So the density function is
\[ - \frac{\partial}{\partial x} \left( \frac{1 - x}{2} \right)^n = \frac{n}{2} \left( \frac{1 - x}{2} \right)^{n-1}. \]
Suppose that $X_i$ are independent copies of a random variable $X$. Let $M_X(t)$ be the moment generating function for $X$. Compute the moment generating function for the average $\sum_{i=1}^n X_i/n$ in terms of $M_X(t)$ and $n$. 
Write $Y = \sum_{i=1}^{n} X_i / n$. Then

$$M_Y(t) = E[e^{tY}] = E[e^{t\sum_{i=1}^{n} X_i / n}] = (M_X(t/n))^n.$$
Suppose $X$ and $Y$ are independent random variables, each equal to 1 with probability $1/3$ and equal to 2 with probability $2/3$.

- Compute the entropy $H(X)$.

- Compute $H(X+Y)$.

- Which is larger, $H(X+Y)$ or $H(X,Y)$? Would the answer to this question be the same for any discrete random variables $X$ and $Y$? Explain.
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- Compute the entropy $H(X)$.
- Compute $H(X + Y)$.
- Which is larger, $H(X + Y)$ or $H(X, Y)$? Would the answer to this question be the same for any discrete random variables $X$ and $Y$? Explain.
$H(X) = \frac{1}{3}(- \log \frac{1}{3}) + \frac{2}{3}(- \log \frac{2}{3})$. 
Entropy — answers

- $H(X) = \frac{1}{3}(-\log \frac{1}{3}) + \frac{2}{3}(-\log \frac{2}{3})$.
- $H(X + Y) = \frac{1}{9}(-\log \frac{1}{9}) + \frac{4}{9}(-\log \frac{4}{9}) + \frac{4}{9}(-\log \frac{4}{9})$.
Entrophy — answers

- $H(X) = \frac{1}{3}(- \log \frac{1}{3}) + \frac{2}{3}(- \log \frac{2}{3})$.
- $H(X + Y) = \frac{1}{9}(- \log \frac{1}{9}) + \frac{4}{9}(- \log \frac{4}{9}) + \frac{4}{9}(- \log \frac{4}{9})$
- $H(X, Y)$ is larger, and we have $H(X, Y) \geq H(X + Y)$ for any $X$ and $Y$. To see why, write $a(x, y) = P\{X = x, Y = y\}$ and $b(x, y) = P\{X + Y = x + y\}$. Then $a(x, y) \leq b(x, y)$ for any $x$ and $y$, so $H(X, Y) = E[- \log a(x, y)] \geq E[- \log b(x, y)] = H(X + Y)$. 
Markov chains

Alice and Bob share a home with a bathroom, a walk-in closet, and 2 towels.
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Each morning a fair coin decide which of the two showers first.
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Each morning a fair coin decides which of the two showers first.

After Bob showers, if there is at least one towel in the bathroom, Bob uses the towel and leaves it draped over a chair in the walk-in closet. If there is no towel in the bathroom, Bob grumpily goes to the walk-in closet, dries off there, and leaves the towel in the walk-in closet.

Problem: describe towel-distribution evolution as a Markov chain and determine (over the long term) on what fraction of days Bob emerges from the shower to find no towel.
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When Alice showers, she first checks to see if at least one towel is present. If a towel is present, she dries off with that towel and returns it to the bathroom towel rack. Otherwise, she cheerfully retrieves both towels from the walk-in closet, then showers, dries off and leaves both towels on the rack.
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**Problem:** describe towel-distribution evolution as a Markov chain and determine (over the long term) on what fraction of days Bob emerges from the shower to find no towel.
Let state 0, 1, 2 denote bathroom towel number.
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Shower state change Bob: 2 → 1, 1 → 0, 0 → 0.

Shower state change Alice: 2 → 2, 1 → 1, 0 → 2.

Morning state change AB: 2 → 1, 1 → 0, 0 → 1.

Morning state change BA: 2 → 1, 1 → 2, 0 → 2.

Markov chain matrix: \[ M = \begin{pmatrix} 0 & 0.5 & 0.5 \\ 0.5 & 0 & 1 \end{pmatrix} \]

Row vector \( \pi \) such that \( \pi M = \pi \) (with components of \( \pi \) summing to one) is \((2/9, 4/9, 1/3)\).

Bob finds no towel only if morning starts in state zero and Bob goes first. Over long term Bob finds no towel \( 2/9 \times 1/2 = 1/9 \) fraction of the time.
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Markov chains — answers

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Bob finds no towel only if morning starts in state zero and Bob goes first. Over long term Bob finds no towel \( \frac{2}{9} \times \frac{1}{2} = \frac{1}{9} \) fraction of the time.
Suppose that $X_1, X_2, X_3, \ldots$ is an infinite sequence of independent random variables which are each equal to 1 with probability $1/2$ and $-1$ with probability $1/2$. Let $Y_n = \sum_{i=1}^{n} X_i$. Answer the following:

▶ What is the probability that $Y_n$ reaches $-25$ before the first time that it reaches 5?
Suppose that $X_1, X_2, X_3, \ldots$ is an infinite sequence of independent random variables which are each equal to 1 with probability $1/2$ and $-1$ with probability $1/2$. Let $Y_n = \sum_{i=1}^{n} X_i$. Answer the following:

- What is the probability that $Y_n$ reaches $-25$ before the first time that it reaches $5$?

- Use the central limit theorem to approximate the probability that $Y_{9000000}$ is greater than 6000.
$p_{-25}25 + p_55 = 0$ and $p_{-25} + p_5 = 1$. Solving, we obtain $p_{-25} = 1/6$ and $p_5 = 5/6$. 

One standard deviation is $\sqrt{9000000} = 3000$. We want the probability to be 2 standard deviations above the mean. Should be about $\int_{2}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx$. 
\[ p_{-25} 25 + p_5 5 = 0 \text{ and } p_{-25} + p_5 = 1. \] Solving, we obtain \[ p_{-25} = 1/6 \text{ and } p_5 = 5/6. \]

One standard deviation is \( \sqrt{9000000} = 3000 \). We want probability to be 2 standard deviations above mean. Should be about \( \int_2^\infty \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \, dx \).
Let $X_i$ be independent random variables with mean zero. In which of the cases below is the sequence $Y_i$ necessarily a martingale?
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- $Y_n = \sum_{i=1}^{n} i X_i$
- $Y_n = \sum_{i=1}^{n} X_i^2 - n$
Let $X_i$ be independent random variables with mean zero. In which of the cases below is the sequence $Y_i$ necessarily a martingale?

- $Y_n = \sum_{i=1}^{n} iX_i$
- $Y_n = \sum_{i=1}^{n} X_i^2 - n$
- $Y_n = \prod_{i=1}^{n} (1 + X_i)$
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- $Y_n = \prod_{i=1}^{n} (1 + X_i)$
- $Y_n = \prod_{i=1}^{n} (X_i - 1)$
Martingales

- Yes, no, yes, no.
If you want *more* probability and statistics...

- **UNDERGRADUATE:**
  1. 18.615 Introduction to Stochastic Processes
  2. 18.642 Topics in Math with Applications in Finance
  3. 18.650 Statistics for Applications

- **GRADUATE LEVEL PROBABILITY**
  1. 18.675 Theory of Probability
  2. 18.676 Stochastic calculus
  3. 18.677 Topics in stochastic processes (topics vary, can be pretty much anything in probability, repeatable)

- **GRADUATE LEVEL STATISTICS**
  1. 18.655 Mathematical statistics
  2. 18.657 Topics in statistics (topics vary, repeatable)

- **OUTSIDE OF MATH DEPARTMENT**
  1. Look up new MIT minor in statistics and data sciences.
  3. Ask other MIT faculty how they use probability and statistics in their research.
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**OUTSIDE OF MATH DEPARTMENT**
(a) Look up new MIT minor in statistics and data sciences.  
(b) Look up longer lists of probability/statistics courses at https://stat.mit.edu/academics/minor-in-statistics/ or http://student.mit.edu/catalog/m18b.html  
(c) Ask other MIT faculty how they use probability and statistics in their research.
Thanks for taking the course!

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- And may the odds be ever in your favor.