18.600 Midterm 1, Spring 2018: 50 minutes, 100 points

1. **Carefully and clearly *show your work* on each problem (without writing anything that is technically not true). In particular, if you use any known facts (or facts proved in lecture) you should state clearly what fact you are using and why it applies.**

2. **No calculators, books, or notes may be used.**

3. **Simplify your answers as much as possible (but answers may include factorials and $\binom{n}{k}$ expressions — no need to multiply them out).**

NAME: ____

1. (20 points) Roll six standard six-sided dice independently, and let X be the number of dice that show the number 6.

(a) Compute the expectation $E[X]$.

(b) Compute the variance $\mathrm{Var}(X)$

(c) Compute $P(X = 5)$

(d) Compute $P(X = 6|X \geq 5)$

2. (10 points) Suppose that E, F and G are events such that

$$P(E) = P(F) = P(G) = .4$$

and

$$P(EF) = P(EG) = P(FG) = .2$$

and

$$P(EFG) = .1.$$

Compute $P(E \cup F \cup G)$.

3. (10 points) Compute the following

(a)

$$\lim_{n\to\infty}(1+5/n)^n$$

(b)

$$\sum_{n=0}^{\infty} 5^n/n!$$

4. (20 points) Suppose that 10000 people visit Alice's new restaurant during its first few months of operation. Each person independently chooses to leave a positive Yelp review (e.g., "Great samosas!") with probability 5/10000, a negative Yelp review (e.g., "Rude servers!") with probability 1/10000 or no review at all with probability 9994/10000. Let X be total number of positive reviews received and Y the total number of negative reviews received.

(a) Compute $E(Y)$ and $\mathrm{Var}(Y)$. (Give exact values, not approximations.)

(b) Use a Poisson random variable to approximate $P(X = 3)$.

(c) Use a Poisson random variable to approximate $P(Y = 0)$.

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5. (10 points) Suppose that a deck of cards contains 120 cards: 30 red cards, 40 black cards, and 50 blue cards. A random collection of 12 cards is chosen (with all possible 12-card subsets being equally likely). What is the probability that this collection contains three red cards, four black cards, and five blue cards?

6. (15 points) Ten people toss their hats in a bin and have them randomly shuffled and returned, one hat to each person. Let X_i be 1 if ith person gets own hat back, 0 otherwise. Let $X = \sum_{i=1}^{10} X_i$ be the total number of people who get their own hat back. Compute the following:

(a) The expectation $E[X]$.

(b) The expectation $E[X_3 X_7]$.

(c) The expectation $E[X_1^2 + X_2^2 + X_3^2]$.

7. (15 points) Bob is at an airport kiosk considering the purchase of a bottle of water with no listed price. Bob is thirsty but is too shy to ask about the price. He happens to know that the price in dollars (denoted X) is an integer between 3 and 7 and he considers each of the values in $\{3, 4, 5, 6, 7\}$ to be equally likely (probability $1/5$ for each). According to Bob's probability measure, find the following:

(a) $E[X]$

(b) $\mathrm{Var}[X]$

(c) $\mathrm{Var}[1.05X]$ (That is, the variance of the sales-tax-inclusive price assuming the airport is in a state with five percent sales tax.)