18.600: Lecture 1

Permutations and combinations, Pascal's triangle, learning to count

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My office hours: Wednesdays 3 to 5 in 2-249

Take a selfie with Norbert Wiener's desk.

Outline

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Politics

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- Natural model for prices: repeatedly toss coin, adding 1 for heads and -1 for tails, until price hits 0 or 100.

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- Let’s start with easier questions.

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- $n \cdot (n-1) \cdot (n-2) \ldots (n-k+1) = n!/(n-k)!$

Permutation notation

- A **permutation** is a function from $\{1, 2, \dots, n\}$ to $\{1, 2, \dots, n\}$ whose range is the whole set $\{1, 2, \dots, n\}$. If σ is a permutation then for each j between 1 and n, the the value $\sigma(j)$ is the number that j gets mapped to.

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- If σ and ρ are both permutations, write $\sigma \circ \rho$ for their composition. That is, $\sigma \circ \rho(j) = \sigma(\rho(j))$.

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- A permutation is "fixed point free" if there are no cycles of length one.

Outline

- Remark, just for fun
- Permutations
- Counting tricks
- Binomial coefficients
- Problems

Outline

Fundamental counting trick

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- This is a useful trick: break counting problem into a sequence of stages so that one always has the same number of choices to make at each stage. Then the total count becomes a product of number of choices available at each stage.
- Easy to make mistakes. For example, maybe in your problem, the number of choices at one stage actually *does* depend on choices made during earlier stages.

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- If you have 5 indistinguishable black cards, 2 indistinguishable red cards, and three indistinguishable green cards, how many distinct shuffle patterns of the ten cards are there?
- Answer: if the cards were distinguishable, we'd have $10!$. But we're overcounting by a factor of $5!2!3!$, so the answer is $10!/(5!2!3!)$.

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- Question: what is $\sum_{k=0}^{n} \binom{n}{k}$?

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- Answer: $(1+1)^n = 2^n$.

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- 366^{23} if repeats allowed. $366!/343!$ if repeats not allowed.