A. FROM TEXTBOOK CHAPTER FOUR:
   2. Theoretical Exercises: 13, 19, 21, 27.

   1. Check that $\text{Cov}(X, X) = \text{Var}(X)$, that $\text{Cov}(X, Y) = \text{Cov}(Y, X)$, and that $\text{Cov} \cdot \cdot \cdot$ is a bilinear function of its arguments. That is, if one fixes one argument then it is a linear function of the other. For example, if we fix the second argument then for real constants $a$ and $b$ we have $\text{Cov}(aX + bY, Z) = a\text{Cov}(X, Z) + b\text{Cov}(Y, Z)$.
   2. If $\text{Cov}(X_i, X_j) = ij$, find $\text{Cov}(X_1 - X_2, X_3 - 2X_4)$.
   3. If $\text{Cov}(X_i, X_j) = ij$, find $\text{Var}(X_1 + 2X_2 + 3X_3)$.

C. Instead of maximizing her expected wealth $E[W]$, Jill maximizes $E[U(W)]$ where $U(x) = -(x - x_0)^2$ and $x_0$ is a large positive number. That is, Jill has a quadratic utility function. (It may seem odd that Jill’s utility declines with wealth once wealth exceeds $x_0$. Let us assume $x_0$ is large enough so that this is unlikely.) Jill currently has $W_0$ dollars. You propose to sample a random variable $X$ (with mean $\mu$ and variance $\sigma^2$) and to give her $X$ dollars (she will lose money if $X$ is negative) so that her new wealth becomes $W = W_0 + X$.
   1. Show that $E[U(W)]$ depends on $\mu$ and $\sigma^2$ (but not on any other information about the probability distribution of $X$) and compute $E[U(W)]$ as a function of $x_0, W_0, \mu, \sigma^2$.
   2. Show that given $\mu$, Jill would prefer for $\sigma^2$ to be as small as possible. (One sometimes refers to $\sigma$ as risk and says that Jill is risk averse.)
   3. Suppose that $X = \sum_{i=1}^{n} a_i X_i$ where $a_i$ are fixed constants and the $X_i$ are random variables with $E[X_i] = \mu_i$ and $\text{Cov}[X_i, X_j] = \sigma_{ij}$. Show that in this case $E[U(W)]$ depends on the $\mu_i$ and the $\sigma_{ij}$ (but not on any other information about the joint probability distributions of the $X_i$) and compute $E[U(W)]$. Hint: first compute the mean and variance of $X$.

4. Read the Wikipedia article on “Modern Portfolio Theory”.
   Summarize what you learned in two or three sentences.