18.440 PROBLEM SET THREE, DUE FEBRUARY 25

A. FROM TEXTBOOK CHAPTER THREE:

1. Problems: 24, 38, 43, 76.
2. Theoretical Exercises: 24.
3. Self-Test Problems and Exercises: 8, 14, 22.

B. Suppose that a fair coin is tossed infinitely many times, independently. Let X_i denote the outcome of the ith coin toss (an element of $\{H, T\}$). Compute the probability that:

1. $X_i = H$ for all positive integers i.
2. The pattern HHTTHHTT occurs at some point in the sequence $X_1, X_2, X_3, \ldots$.

C. Two unfair dice are tossed. Let $p_{i,j}$, for i and j in $\{1, 2, 3, 4, 5, 6\}$, denote the probability that the first die comes up i and the second j. Suppose that for any i and j in $\{1, 2, 3, 4, 5, 6\}$ the event that the first die comes up i is independent of the event that the second die comes up j. Show that this independence implies that, as a 6 by 6 matrix, $p_{i,j}$ has rank one (i.e., show that there is some column of the matrix such that each of the other five column vectors is a constant multiple of that one).