1 Mating of trees and space-filling SLE

1. Consider a stable Lévy process L_t (indexed by $\mathbb{R}$, defined modulo additive constant) with only positive jumps and with parameter $\alpha \in (0, 1)$. Let $J_1, J_2, \dots$ be an enumeration of the jumps. For each vertical jump line segment J_i, which occurs at a time t_i, let T_i denote the next subsequent time after t_i at which the process $L.$ reaches the value of L before the jump (i.e., the left limit of $L.$ at t_i). Then denote by S_i the set of times at which L (restricted to $[t_i, T_i]$) achieves a minimum value. That is,
$$S_i = \{t \in [t_i, T_i] : s \in [t_i, t] \text{ implies } L_s \leq S_t\}.$$
The collection $\{S_i\}$ is a random countable collection of pairwise disjoint closed subsets of $\mathbb{R}$ that are "nested" in a certain sense. Now suppose that $\{\tilde{S}_i\}$ is an independent instance of this random collection. Declare an element of $\{S_i\}$ and an element of $\{\tilde{S}_i\}$ to be adjacent if they intersect. Now we have a random bipartite graph. Is this bipartite graph almost surely connected? This problem is equivalent to a problem about the connectedness of graph of complementary components of a self-intersecting but not space-filling SLE curve (two components connected if boundaries intersect).

2. Could we at least prove connectedness if we had k independent instances of $\{S_i\}$ (for some sufficiently large finite value k) instead of 2?

3. Mullin's bijection implies that uniform-spanning-tree-decorated random planar maps scale have a certain scaling limit in the peanosphere topology. Can this be generalized to UST-decorated quadrangulations, UST-decorated triangulations, or other types of UST-decorated discrete graphs?

4. Can one further generalize the peanosphere convergence for FK models (as derived via the hamburger-cheeseburger bijection) to triangulations, quadrangulations, etc.?

5. In hamburger cheeseburger setting, suppose we choose independent instances of FK given planar map. Can one show that in limit of the coupling the surface is the same with curves different.

6. Can we actually check that the various flow lines in SLE_{12} correspond to analogous GFF flow lines in the field generated by SLE_{12} through the imaginary geometry story?

7. Explain Brownian motion results one obtains as consequence of mating of trees (alternating directions, getting coupling of flow line and field, what path determined by field means on Brownian motion side).

8. Can one put metric on peanosphere when $\gamma^2 \neq 8/3$? First prove that an exponent exists where number of ϵ steps needed scales like ϵ to that power. How about a sort of KPZ like formulation, where one weights paths by length?

9. Neinhuis conjecture of 6-vertex variance and c.

2 Liouville Quantum Gravity Properties

1. Almost sure commutativity of coordinate change and pass from measure to field (over class of all possible coordinate changes).

2. Understand the torus. The LQG torus defined via resampling. The LQG torus defined in terms of some Laplacian determinant. The Brownian map torus. The peanosphere torus. How are they all defined and how are they all related? Can something be said about some of these at least in the $\gamma^2 = \kappa = 2$ UST case? Or the $\gamma^2 = \kappa = 8/3$ case?

3 KPZ growth

1. Discrete KPZ: another interpolation between (-1)-DBM and 0-DBM

2. KPZ holds for KPZ: that is, one can consider the SRW measure on paths, then weight randomly by white noise to get a new measure, and the new measure satisfies a KPZ. Take distance between two walks to be 2^{-k} where k is last point at which they agree. You can think about covering your random subset of the set of all paths — either fixed radius in this standard metric, or of fixed volume in the random measure.

3. Can one characterize KPZ growth by scale invariance and local independence?

4 Other SLE stories

1. Consider SLE_6 on universal cover of complement of grid of points. This process has a boundary that looks like SLE_6 boundary, but in another sense it approximates Brownian motion. What good are these sorts of "in between Brownian motion and SLE_6" objects?

2. What kind of convergence story is there for the $O(n)$ on a random planar map? Is there any sense (analogous to peanosphere sense, say) in which converge is provable?

5 Other interesting models

1. Scaling limits for tricolor percolation

2. Dynamical triangulations and causal quantum gravity.

3. Understand the Gaussian field arising from commutative Yang Mills theory (electromagnetism).