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of Letters, Pencils, Noncommutative Geometry

65, June 2014
3 non-degenerate critical points
$W \left( \frac{t}{v}, \frac{t}{v}, \frac{t}{v} \right) \cap D \times (\mathbb{C} \setminus \{0, \infty\}) = \emptyset$.

$B = 3$ points

3 non-degenerate
$W \cap D = \emptyset$.

$\frac{t}{v} = \frac{t}{v}$ isometric curve (torus)

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$\left( \frac{t}{v}, -2 \frac{t}{v} \right)$ - curves

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deformation of $X_0$.

Def. of a formal one-parameter

Dirichlet $\mathbb{E}_p$ ($\mathfrak{p} = 0$).

Dirichlet near $\mathfrak{p} = 0$.

According to [1][2][3], namely, the fibre

of the pencil over a formal

scheme $O_{X_0}$.

The category of perfect complexes

$\text{Perf}(X_0) \subset \mathcal{D}(X_0)$, the

category of coherent sheaves

$\mathcal{D}(X_0)$, the bounded derived

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Homological mirror symmetry

Homological mirror symmetry
\[ \mathcal{D}(X) \text{ or } \mathcal{D}(X \setminus \{ \infty \}) \quad \text{as } \infty \to \infty \]

\[ 0 = \partial_0 = \partial_1 = \cdots \quad \text{or } 0 = \bar{\partial}_0 = \bar{\partial}_1 = \cdots \]

In more cases, certain formal \( \mathcal{D}(M) \), the Landau-Ginzburg \[ \mathcal{D}(X \setminus \{ \text{pt} \}) \]

\[ \mathcal{F}(X) \text{ with } \text{bulk parameters} \]

\[ M(X, \infty) \quad \text{or} \quad \text{suitable } \mathcal{F}(X) \]

\[ \text{with bulk parameters }\]

\[ \text{wrapped }\]

\[ \text{for a suitable } \mathcal{F}(X) \]

\[ \text{or a suitable } \mathcal{F}(X) \]
Theorem \cite{[g]}. The deformation of $X_{\infty}$ is a formal one-parameter dicr near $x \in \mathbb{P}^2 (\mathbb{C})$ ($b = 0$).

Theorem \cite{[g]}. The deformation of $X_{\infty}$ is a scheme.

Category: The parameter $g$ counts $f (X, g)$. The relative Fukaya category of the fiber minus base locus.

Category of perfect complexes.

Category of coherent sheaves.

Theorem \cite{[g]}. The Fukaya category is contractible to $\mathcal{M}$.

Theorem \cite{[g]}. The Fukaya category associated to $\mathcal{M}$ is contractible to $\mathcal{D} (X, g)$. The bounded derived category of coherent sheaves.
In most cases, certain formal completions are necessary, which we have omitted.

Let $X$ be a scheme over $\mathbb{Z}$. The Landau-Ginzburg\footnote{Formal parameter} $W = \text{wrapped} \ (X \setminus \text{sing})$, where $\text{sing}$ is the singular locus of the generic fiber of the deformation. 

\[ f(x) = \prod_{\ell \mid m} (1 - x^{\ell}) \]
over ĝP.

This extends to a pencil.

Ignoring the fact that it only uses \( W : X \rightarrow \infty \)

(symphlectic topological terms,

\( \nabla \)-pencil in \( X \times \mathbb{R} \), not the

This part only uses

5

\( \Gamma \)

the categories we have

pencil itself, but none of

Conventionally, it is the

What is the central object?

\( \sigma \) determines each other

structure do not really

The different parts of the

Fundamentally unsatisfactory

The picture in the previous slide is

Problem
The blowdown map \( \pi \) is:

\[ \pi^{-1}(x, y) = \text{pullback of } \pi^{-1}(x, y) \text{ via } \]

\[ \text{It still comes with } \psi : x \mapsto y. \]

For generic \( z \),

\[ x \neq \infty \]

for branch point. Hence,

\[ \text{cover (coefficient not a } \text{genus degree } 3 \text{ branch point).} \]

A branched cover (in the manner of Abouzaid–Auroux–Coxon–Katzarkov–Kontsevich...).
\[(\mathbb{C} - \xi)(-x_1^3 + x_2x_3 + x_4) = (\mathbb{C} - \xi)(x_1^3 + x_2x_3 + x_4^2 + M - \mathbb{C})\]

A priori, this + further periodicity

we have
\[\text{Sing}(\varphi) = \varphi(\text{point}) \neq \varphi(\text{point})\]

\[\text{for } (\xi, \mu) \text{ non-reducible for}\]

is semireducible

\[\varphi(\text{point}) = \varphi(\text{point})\]

\[\text{relations}\]
\[\text{relations with relations}\]
\[\text{is described by a}\]
\[\varphi(\mathbb{M} : \varphi)\]

\[\text{Cartan\'s relationship}\]
This question extends to other \textit{in Landau-Ginzburg theory}.

In which sense can the previous algebraic-geometric framework be thought of as a pencil?

\textit{In geometry (e.g. noncommutative)}

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\begin{pmatrix}
\begin{array}{c}
\small\text{noncommutative}
\end{array}
\end{pmatrix}
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\textit{deformation} of \textit{algebraic}.

\textit{This question extends to other}.

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\textit{Difficulties and graded sheaves}:
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\begin{pmatrix}
\begin{array}{c}
\small\text{Difficulties and graded sheaves}:
\end{array}
\end{pmatrix}
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\textit{Introduction}.

\textit{A graded sheaf structure on a dg scheme, sheafification, and line bundles} (\textit{X, l}) gives rise to \textit{previous algebro-geometric}

\textit{Given on algebraic variety} \textit{X}.

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\begin{pmatrix}
\begin{array}{c}
\small\text{Given on algebraic variety} \textit{X}.
\end{array}
\end{pmatrix}
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Zero extension is not necessarily a square by the dg bimodule $P$, but an extension of the dg $A$-module $P$. Then $G$ is left and right exact as an $A$-bimodule, as well as an $A$-module. If $\psi$ is a replacement for $\phi$, then $\psi$ is a $\mu$-algebra structure on $\phi$. Don't know about $\mu$-structures.

(For $\otimes$ is an algebraic structure on an $A$-bimodule which is not invertible.

Let $A$ be an $\infty$-algebra, and $P$ a commutative (algebraic) geometry.
localization

equivalent to a suitable
distinction construction

The complement of the

(refer to "localization theory"

structure can be analyzed through

the entire

we call the first order part of the

\[ \text{Hom}_A(A', P_1) \xrightarrow{\phi} \text{Hom}_A(A, P_1) \]

whose boundary map

Consider \( P \) as an \( A \)-bimodule.

Analyzing a noncommutative divisor

\[ \phi \]
The $\mathcal{O}$'s are generated algebraic pencils, a notion dual to a \underline{scheme} (in a category algebraic geometry). Given rise to a \underline{surface} (in algebraic geometry).

Example: Any pencil of hyperplanes.

We now add the condition that schemes are preserved by differentiation, and we define a family of algebraic varieties.

The $\mathcal{O}$'s induce a notion of degree, a notion of grading, and we consider $\mathcal{O}$ as a dg category. The $\mathcal{O}$'s are generated algebraic pencils, a notion dual to a scheme.

Definition: A noncommutative pencil

\[ \mathbb{D} \quad \mathcal{O} \quad \mathcal{X} \left[ \mathcal{E}_\mathcal{O} \otimes \mathcal{E}_\mathcal{X} \right] \left( \mathcal{Y} \right) \]

\[ \text{Pencil is a collection of maps} \]

\[ \text{such that the weights and} \]

\[ \text{where one considers the grading,} \]

\[ \text{set } \mathcal{V} \rightarrow \mathcal{D}, \quad M \in \text{Hom}(\mathcal{V}, \mathcal{C}), \text{ let's} \]

\[ \text{Noncommutative pencils} \]
Laudau–Ginzburg model.

Associated noncommutative
tate degree 2, called the
t has a formal deformation.

Take $R = \mathbb{C}[[t]]$, (2 = \infty \mathbb{C}_d)$. (can $A$-algebra over $\mathbb{C}[[t]]$)

t a deformation $R = \mathbb{C}[[t]]$ of $\mathbb{C}_d$, formal disc around $\mathbb{C}_d$.

"Point", for example, a

First, one can also take

$G_0 : \mathbb{C}_d \to A$.

The two bimodule maps

are $\mathbb{C}_d$, or equivalently $(\mathbb{C}_d)$.

We have a bimodule map

After taking a noncommutative

pencil,
The Abbe at $\infty$ counterclockwise through a reflection normal to C.

Let $F$ be a smooth curve through $\infty$.

What makes up the pencil?

Pencils.

Canonical with a noncommutative
Theorem $\mathfrak{a} = \mathfrak{a}^2(M)$ can be equipped

with exact
$L \mathfrak{a} (\mathfrak{X}\mathfrak{l}(\mathfrak{\infty}))$

Let $\mathfrak{a}$ be a smooth
smooth

Penalties for $\mathfrak{L} = \mathfrak{L}$, the "first fiber"

a symplectic leaf

X a symplectic manifold which

Symplectic geometry
Variables, get $f^2(X)$

obvious change of $f(X)$. After an

close relative of $f$ corresponds to (a

Let model on $A \mathbb{G}$
The noncommutative

Category $G(X \setminus X)$

the wrapped $T$- 된다?

$A \mathbb{G}$ corresponds to

$
\begin{array}{l}
X \rightarrow \mathbb{G}^T \\
\text{associated to}
\end{array}$

Noncomm. pencil

$F(X \setminus X)$

corresponds to

$F(\infty, \mathbb{G})$

Para of relationships. Ties have no geometric meaning?
Accordingly, it needs to be adapted algebraically for the situations, but the we can consider other algebraic forms. In this case, many could be singular.

Algebraic pencils:

Could be singular

$L = K^I$, $x_0$

Homological mirror symmetry for pencils