Lefschetz fibrations in symplectic topology

Part IV

I. Classification
II. Stabilization
III. Parity

 Inspiring gentleman

 ... and of course this

 Murphy

 Groups - Paradox, Ragingity,
of Anatomy, Bog Km,

 largely based on work

 Real, Real, MT
0 BACKGROUND
Cycles are transversal. The vanishing cycles of dimension $2n$. The vanishing cycles of dimension $2n-2$ of a manifold of dimension $2n-2$, of a manifold of dimension $2n$. If now a compact symplectic manifold of dimension $2n$.

Artinian dimension.

$\text{Artinian dimension}$.

The boundary monodromy is $\text{(v, v)} \rightarrow \text{(v, v)} \rightarrow \text{(v, v)} \rightarrow \text{(v, v)}$.

Homology collapses.

After changing factors, critical point, $\perp (x, x^2) = x^2 + x^2$.

Compact non-compact complex modelled on the simplest compact complex manifold with many critical points.

$n$ has finitely many critical points.

where the fibre $F$ is an oriented smooth 2-dim. disc.

We consider the low-dimensional situation.
which we can't solve, obtusely.

I classification issues.
\[ a - \delta + (\Lambda | \bar{n})^{2} = a + q_{1}a_{1}^{2}a_{2}a_{1}^{2} - q_{1}a_{1}^{2} - q_{1}a_{1}^{2}a_{2}a_{1}^{2} \]

Hence, its trace is

\[ 2 \cdot \begin{pmatrix}
\frac{1}{2} & \frac{1}{2} \\
\frac{1}{2} & -\frac{1}{2}
\end{pmatrix}
\]

\[ \begin{pmatrix}
\frac{1}{2} & \frac{1}{2} \\
\frac{1}{2} & -\frac{1}{2}
\end{pmatrix} \begin{pmatrix}
\frac{1}{2} & -\frac{1}{2} \\
-\frac{1}{2} & \frac{1}{2}
\end{pmatrix} = \begin{pmatrix}
1 & 0 \\
0 & 1
\end{pmatrix}
\]

\[ \begin{pmatrix}
1 & 0 \\
0 & 1
\end{pmatrix} \cdot \begin{pmatrix}
0 & 1 \\
1 & 0
\end{pmatrix} = \begin{pmatrix}
1 & 0 \\
0 & 1
\end{pmatrix}
\]

The boundary monodromy is an invariant. The boundary monodromy is an invariant.

\[ \tau \in \text{SL}(2, \mathbb{Z}) \cong \text{Aut}(\mathbb{Z}^{2}) \]

The (primitive) homology classes, closed curves are determined by

\[ H_{1}(F) \cong \mathbb{Z}^{2}, \quad \text{nonseparating} \]

\[ \tau \in \text{Aut}(\mathbb{Z}^{2}) \]

Simplest example (Auroux 2013).

\[ F \]
These are actually conjugate, and the
\[
\begin{pmatrix}
-2 & 1 \\
-1 & 1 \\
\end{pmatrix}
\begin{pmatrix}
0 & 1 \\
-1 & 0 \\
\end{pmatrix}
\begin{pmatrix}
-2 & 1 \\
-1 & 1 \\
\end{pmatrix}
\begin{pmatrix}
0 & 1 \\
-1 & 0 \\
\end{pmatrix}
\sim
\begin{pmatrix}
0 & 1 \\
-1 & 0 \\
\end{pmatrix}
\begin{pmatrix}
0 & 1 \\
-1 & 0 \\
\end{pmatrix}

\begin{pmatrix}
0 & 1 \\
-1 & 0 \\
\end{pmatrix}
\sim
\begin{pmatrix}
0 & 1 \\
-1 & 0 \\
\end{pmatrix}
\begin{pmatrix}
0 & 1 \\
-1 & 0 \\
\end{pmatrix}
\end{array}
\]

Example: \( A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \)

Furthermore, by boundary monodromy.

distinguished by boundary monodromy.

it could still be the case that they are

distinguished by the fact that they are,

This has \( \frac{a}{\phi(b)} \) orbits, which are

This has \( \frac{a}{\phi(b)} \) orbits, which are

\[ \frac{a}{\phi(b)} \]

Thus, \( a \to \frac{a}{\phi(b)} \).

There is \( \frac{a}{\phi(b)} \) on \( \frac{\pi}{2} \).

orientation change

\( A \to \begin{pmatrix} 0 & 9 \\ -9 & 0 \end{pmatrix} \) \( \sim \)

\( \begin{pmatrix} 9 & 0 \\ 0 & -9 \end{pmatrix} \)

\( A \leftarrow \begin{pmatrix} 9 & 0 \\ 0 & -9 \end{pmatrix} \)

plus \( \text{GL}_2(\mathbb{Z}) \)-action is

only mod \( b \). The Hurwitz move

where \( a \equiv b \equiv \text{det}(A) \) and \( a \) matches

\( A = \begin{pmatrix} 0 & 9 \\ -9 & 0 \end{pmatrix} \)

\( A = \begin{pmatrix} 9 & 0 \\ 0 & -9 \end{pmatrix} \) \( \sim \)

commute.

one has

After left multiplication with \( \text{GL}_2(\mathbb{Z}) \),

important of our Legendre function.

However, \( \text{det}(A) \) is not a complete
In particular, the previous two examples.

\[
\begin{array}{c|c|c}
\hline
1 & 2 & 3 \\
\hline
2 & 3 & 1 \\
\hline
3 & 1 & 2 \\
\hline
\end{array}
\]
However, the classical (differentiable) cover of \( \mathbb{R}^3 \) or a double branched is a closed higher dimensional idea: think Lefschetz.

\[
\text{Lefschetz: Sphere} \quad \mathbb{R}^3 
\]

Now, take intersecting two branched points is a closed.

The preimage of a path in \( \mathbb{R}^3 \) is a cover of \( \mathbb{R}^3 \) or a double branched.

\[ \bigcup_{\mathbb{R}^3} = \bigcup_{\mathbb{R}^3} \]

\[ \text{po} = \mathbb{R}^3 \]
The symplectic manifold \( \tilde{\text{Symplectic Manifold}} \) is defined by the equation \( E = \mathbb{R}^2 \times \mathbb{R}^3 \).

Total space \( E \cong \{ x^2 + y^2 = 1 \} \times \mathbb{R}^3 \) (called a \( \text{Generalized Surface} \)).

Intersection form (4):

\[ \langle x, y \rangle = 1 \]

Total space \( E \cong \{ x^2 + y^2 = 1 \} \times \mathbb{R}^3 \) (no cancellation possible in this case).

Intersection form (5):

\[ \langle x, y \rangle = 1 \]

Total space \( E \cong \{ x^2 + y^2 = 1 \} \times \mathbb{R}^3 \) (intersection form is trivial).

Intersection form (6):

\[ \langle x, y \rangle = 1 \]

Total space \( E \cong \{ x^2 + y^2 = 1 \} \times \mathbb{R}^3 \) (intersection form is trivial).

\( F = \{ \text{No minor rank, 3 vanishing cycles} \} \)

Same example in higher dimension.
homomorphic invariant

the total space $E$ is a principal bundle

choice of vanishing cycles

if the first $m$ are always $S^3$

but what if $E$ then? the underlying simplicial

vanishing cycle, $E \neq \mathbb{R}^3$

for all $m+1$ points

$$f = \frac{1}{2} x_1 + \frac{1}{2} x_2 + \frac{1}{2} x_3 \quad x_{m+1}$$

we get total space $E \cong \mathbb{T}^{m+3}$
The opposite seems much more likely.

Then, with "very high probability," the

a chain:

endpoints of the associated path form

such that the

in

Choice of non-vanishing cycles in

Conjecture (Seidel, 2006) Pick a random

topology:

The most ill-posed Conjecture in symplectic

Corollary: The affine complex 3-fold

map by a

+ b points

vanishing cycles corresponds to a choice of

Thus

\[ \{ x, y \in \mathbb{R}^2 \mid x^2 + y^2 = 1 \} \]

For (compare 4.6):

Question How about the following:

are all symplectically isomorphic.

Conjecture
Denis Anselm understands, even which possibly no one other than in the lowest dimension...
The idea behind Ahu"rt's theorem is:

\[
\begin{array}{c}
\text{connected sum:} \\
\text{can be thought of as a\ Frie} \\
\text{fixed pattern of vanishing cycles, and} \\
\text{Friequlation process adds a} \\
\end{array}
\]

The \text{Friequlation process adds a boundary monodromy cycle.}
Let's consider a total preclusion of a theorem (Gruen-Pardan).

Any given domain is the sequence (embedded into one belonging to our category, can be symplectically embedded) of axiom given domains, such that every contradiction there is an explicit sequence.

Contrary, there is an explicit sequence into one belonging to that sequence. Any other preclusion can be embedded of axiom of Richardson's preclusions, such that
Recent progress in low-dimensional care (Baghurst and collaborators)
Classical low-dimensional situation

**Theorem** let \( F \) be a surface (as usual, compact, oriented, \( \partial F \neq \emptyset \)). In \( \text{Diff}(F, \partial F) \), a product of Dehn twists is never isotopic to the identity.

**(Smith, 1999)**

**Proof sketch** for \( g(F) \geq 2 \). Take the closed surface \( F \subseteq S \), \( * \in S \setminus F \), and

\[
\pi_0(\text{Diff}(F, \partial F)) \rightarrow \pi_0(\text{Diff}(S, *))
\]

pass to universal cover, extend to boundary at \( * \)

Dehn twists "turn \( S^1 \) to the right".

Arbitrary dimensions

**Theorem (S, unpublished)** let \( F \) be a Liouville domain (assume for simplicity that \( H^4(F, \mathbb{Z}) = 0 \)). In \( \text{Symp}(F, \partial F) \), a product of Dehn twists is never isotopic to the identity.

**Proof sketch** suppose the contrary, \( \tau_V \neq \tau_W = \text{id} \).

Then, the \( V_i \) are vanishing cycles for a Lefschetz fibration that extends over the two-sphere, \( \pi: E \rightarrow S^2 \).

By looking at pseudo-holomorphic sections, this can't have critical points.
Conjecture. The same is true for long, higher-dimensional, isolated Donaldson invariants. The same boundary monodromy and with arbitrarily thin $b_2(E)$ and

Corollary. We get further filtrations. Hence $h(-1) \phi (\phi^{-1})_0 = \phi$.

Theorem (Biqueli-Harnack). Its conjugate is $\phi$. Any Dehn twist $t$.

Let have a homomorphism of $\mathbb{Z}$. Then $f = \phi$, $\mathbb{Z}$. Quantitative positivity in this case.