KRISTEN HENDRICKS
ROBERT UPTIGHT + DAVID TREUNAN

Also includes discussion of work of:

PAUL SEidel, MTT

SYMPLECTIC FIXED POINTS
STEINROD QUANTERS AND
can be used in a reference

OXFORD, OCTOBER 2014
For any many periodic points, there are no finite periodic points.

In general, yes, e.g., irrational points (Zhang - Gieser - Hei).

Inaccessible (Zhang - Gieser - Hei):

(un必要的) (Zhang - Gieser - Hei)

perp to the line, there must be an infinite number of periodic points. (x)

in principle to id (symplecticity). If

where preserves the symplectic form,

Let \( \phi : \mathbb{R}^n \to \mathbb{R}^n \) be a diffeomorphism.

(a symplectic form \( \omega \) is \( \omega = d\gamma \) \( + \cdots \) \( \gamma \) de = 0.

Let \( M \) be a closed manifold with

Symplectic diffeomorphisms.
A symplectic diffeomorphism, the local parametric since \( \mathcal{L} \) comes with

\[
\mathbf{H}_{1}(x, \theta) = H_{1}(\theta(x), \theta(\theta(x)), \theta(\theta(\theta(x))))
\]

and its flow \( (e_{t}) \). Suppose \( \dot{x}_{0} = 0 \).

Given \( M \) and \( H \in C^{1}(\mathbb{R}^{2n}) \), consider maps \( \mathbf{H} \) of Hamiltonian systems.

Historical origin: "Poincaré return"
map between them.

Of vector spaces with an involution

Roughly, this is the theory of pairs

\[ \left( p, q \right) \in \mathbb{V} \times \mathbb{V} \]

non-contradicted

Given by an involution with an action of the group \( \mathbb{Z}/2 \)

A vector space over a field \( \mathbb{K} \)

\[ \text{char}(\mathbb{K}) = 2 \text{ is nontrivial.} \]

More concretely, we

\[ \varphi \circ \varphi = \text{id} \]

\[ \text{a } \mathbb{K}\text{-admissible form a semisimple algebra category.} \]

\[ \text{char}(\mathbb{K}) = 2 \text{ is boring; vector spaces} \]
Take cohomology first.

\[ H^k(R) \cong \text{ker}(d) \wedge \text{coker}(d) \]

Example: Trivial action:

\[ H^k(\mathbb{Z}) = \mathbb{Z} \wedge \mathbb{Z} \]

\[ H^k(\mathbb{Z}/2\mathbb{Z}) = \mathbb{Z}/2\mathbb{Z} \wedge \mathbb{Z}/2\mathbb{Z} \]

The group cochain complex is the formal variable \( u \) which, in the negative powers, may have formal power.

\[ \text{Group cohomology} \quad \text{char}(k) = 2 \quad \text{from} \]

\[ u^2 = 0 \quad \text{in degree} \ 1, \]
Note that $H^r(\mathbb{Z}/2\mathbb{Z}; \mathbb{V} \otimes \mathbb{V})$ is additive in $\mathbb{V}$, but the underlying chain complex isn't.

After tensoring with $K(\mathbb{V})$, it induces (x).

Consider the Tate map on cohomology.

This is not additive, but the induced map on cohomology becomes additive after multiplying with $u$. Then the two factors, $\mathbb{V}$ sends $\mathbb{V}$ to $\mathbb{V} \otimes \mathbb{V}$.

Consider the Tate map $v(\mathbb{V})$ with the $\mathbb{V}$-action which exchanges $\mathbb{V}$ and $\mathbb{V}$. Hence the Tate map.

$H^r(\mathbb{Z}/2\mathbb{Z}; \mathbb{V} \otimes \mathbb{V}) \cong \text{Sym}^r(\mathbb{V}) \oplus \mathbb{V}([r])$.
Example \( (\Leftrightarrow \oplus \text{ bounded, each } \cap \text{ finite}) \)

\[ E = \mathcal{H}(\mathcal{V}, \mathcal{W}) \text{ as a graded vector space} \]

Specific sequence.

Assuming \( E \) is bounded, the finite

\[ \vdots \]

\[ E_2 = \mathcal{H}(\mathcal{W}) \]

\[ E_1 = \mathbb{C}(\mathcal{W}, \mathcal{V}) \]

Take \( (\Leftrightarrow \oplus \text{ bounded, each } \cap \text{ finite}) \)

Take \( (\Leftrightarrow \oplus \text{ bounded, each } \cap \text{ finite}) \)

The resulting \( H^*(\mathbb{C}, \mathcal{V}) \text{ (for } * \text{)} \) are quasi-isomorphism invariants.

\[ \partial_{\mathcal{V}} = d + \mathbb{C}(\mathcal{W}, \mathcal{V}) \]

by the Tate map on cochains.

\[ \mathcal{H}^*(\mathbb{C}, \mathcal{V}) \text{ not graded} \]

Example \( (\Leftrightarrow \oplus \text{ bounded, each } \cap \text{ finite}) \)

Chain complexes with involution


degree \( (\Leftrightarrow \oplus \text{ bounded, each } \cap \text{ finite}) \)
Similarly, Tate version.

\[ H^*_T(M) := \text{cochains with } \text{equivariant cohomology} \]

**Example:** Given any \( M \), take \( H^*(M) \).

- If \( M \) is fixed-point free, \( H^*_E(M) = 0 \).
- \( \text{In fact, } H^*_E(M) = H^*(M/\mathbb{Z}_2) \).

**Second spectral sequence implies:**

\[ H^*_E(M) \cong H^*(M) \otimes \mathbb{Z}_2 \]

**Not degree preserving (Take map):**

\[ \gamma \text{ H}_E(M) \otimes \mathbb{Z}_2 = (\text{H}_E(M) \otimes \mathbb{Z}_2) \]

- If \( p \) is fixed-point free, \( H^*_E(M) = 0 \).

**First spectral sequence:**

\[ E^1_{p, q} = \text{H}_p(M, \mathbb{Z}_2) \]

\[ E^2_{p, q} = \text{H}_p(M, \mathbb{Z}_2) \]

**Not degree preserving:**

\[ \gamma \text{ H}_2(M) = \text{H}_2(M) \]

**Also:**

\[ H_2(M) \cong \text{H}_2(M) \cong \text{H}_2(M) \otimes \mathbb{Z}_2 \]

**Differential on \( E^2 \):**

\[ d = p - \text{id} \]

**Fiberwise:**

\[ E^1_{p, q} = \text{H}_p(M, \mathbb{Z}_2) \]

**Fiberwise Tate version:**

\[ H^*_T(M) := \text{cochains with } \text{equivariant cohomology} \]

**K-theoretic coefficients:**

\[ H^*_E(M) := \text{K-theoretic cochains with } \text{equivariant cohomology} \]

**Not degree preserving:**

\[ \gamma \text{ H}_E(M) = \text{H}_E(M) \]

**Second spectral sequence implies:**

\[ H^*_E(M) \cong H^*(M) \otimes \mathbb{Z}_2 \]

**Not degree preserving:**

\[ \gamma \text{ H}_2(M) = \text{H}_2(M) \]

**Example:** Given any \( M \), take \( H^*(M) \).

- If \( M \) is fixed-point free, \( H^*_E(M) = 0 \).

**Second spectral sequence implies:**

\[ H^*_E(M) \cong H^*(M) \otimes \mathbb{Z}_2 \]

**Not degree preserving:**

\[ \gamma \text{ H}_2(M) = \text{H}_2(M) \]

**Example:** Given any \( M \), take \( H^*(M) \).

- If \( M \) is fixed-point free, \( H^*_E(M) = 0 \).

**Second spectral sequence implies:**

\[ H^*_E(M) \cong H^*(M) \otimes \mathbb{Z}_2 \]

**Not degree preserving:**

\[ \gamma \text{ H}_2(M) = \text{H}_2(M) \]

**Example:** Given any \( M \), take \( H^*(M) \).

- If \( M \) is fixed-point free, \( H^*_E(M) = 0 \).

**Second spectral sequence implies:**

\[ H^*_E(M) \cong H^*(M) \otimes \mathbb{Z}_2 \]

**Not degree preserving:**

\[ \gamma \text{ H}_2(M) = \text{H}_2(M) \]
Theorem theorem applied
This is a consequence of the
The induced map
Boothby

More precisely, for \( x \in H^1(M) \)

\[ (y) \mapsto \int_M (y) \]
Example $k$-fold Dehn twist on annulus.

Then $\text{HF}^*(\phi) \cong \text{HF}^*(\mathbb{R})$.

Take the flow $\phi_f$ for small $t > 0$.

$K|f|n$ positive in outward direction.

$|infty| \cong |infty| \cong \text{HF}(\mathbb{R}_n)$ with $|\text{HF}| = \text{cont}$.

(The class of $\text{per, i.e.,}$ $\phi_f$)

The $|\text{HF}|$ is invariant under isotopy.

The fixed points are nondegenerate.

If $|\text{HF}|(\phi) \neq 0$, then $|\text{HF}|(\phi)$ is the

Frobenius number is equal to $|\text{HF}|(\phi)$.

As $|\text{HF}|(\phi) = \text{Frobenius number}$.

Example $k$-fold Dehn twist on annulus.

Then $\text{HF}^*(\phi) \cong \text{HF}^*(\mathbb{R})$.

Take the flow $\phi_f$ for small $t > 0$.

$K|f|n$ positive in outward direction.

$|infty| \cong |infty| \cong \text{HF}(\mathbb{R}_n)$ with $|\text{HF}| = \text{cont}$.

(The class of $\text{per, i.e.,}$ $\phi_f$)

The $|\text{HF}|$ is invariant under isotopy.

The fixed points are nondegenerate.

If $|\text{HF}|(\phi) \neq 0$, then $|\text{HF}|(\phi)$ is the

Frobenius number is equal to $|\text{HF}|(\phi)$.

As $|\text{HF}|(\phi) = \text{Frobenius number}$.

Example $k$-fold Dehn twist on annulus.

Then $\text{HF}^*(\phi) \cong \text{HF}^*(\mathbb{R})$.

Take the flow $\phi_f$ for small $t > 0$.

$K|f|n$ positive in outward direction.

$|infty| \cong |infty| \cong \text{HF}(\mathbb{R}_n)$ with $|\text{HF}| = \text{cont}$.

(The class of $\text{per, i.e.,}$ $\phi_f$)

The $|\text{HF}|$ is invariant under isotopy.

The fixed points are nondegenerate.

If $|\text{HF}|(\phi) \neq 0$, then $|\text{HF}|(\phi)$ is the

Frobenius number is equal to $|\text{HF}|(\phi)$.
Note: Differentiation of matrices.

\[
\text{Given: } f(x) = (x - c)(x - d)
\]

where the \(x\)-grading is determined.

\[
f'(x) = x - \alpha \in D.\text{ a complex Hered chain}
\]

Formally (Morse). Define \(A\) is not a nondegenerate critical point iff there is no \(m \neq 0\) such that \(A\) is a critical point of the vector field \(\partial_l\).

Assume from now on that the fixed points of \(A\) are critical points of \(A\).

\[
A(x) = -\int_0^1 \lambda_x + \mathcal{C}(\lambda(1))
\]

A function \(A: I \times \mathbb{R} \to \mathbb{R}\)

\[
\int I \times \mathbb{R} \to \mathbb{R}
\]

Turned loop space
and its cohomology is $H^q(F)$.

$\delta g = d_0 + d_1 + \ldots + d_q + \ldots$

$CF^g(F) = CF^g(F) \oplus \mathbb{Z}$

If $g \neq h$, one defines an homotopy action of $\mathbb{Z}$ on $CF^g(F)$. An homotopy action which commutes a homotopy action.

Let $\phi = d_0 + d_1 + \ldots + d_q$.

is an infinite sequence.

Since $\phi$ is an infinite sequence, the action of some chain homotopy $\phi$ is such that

$p = \phi(t + \frac{T}{2}) = \phi(t) + \frac{T}{2}$

satisfies

$p - \frac{T}{2} = \phi(t + \frac{T}{2}) - \phi(t) = \phi(t + \frac{T}{2} - t) = \phi(T - \frac{T}{2})$


The space $\mathcal{A}_g$ cannot be moved.

Equivariant Floer cohomology

Invariance of the action

$\tau = \phi(t)$}

The $T$-invariance of loops:

The space $\mathcal{A}_g$ cannot be moved.
mythical.

and the differences are a pun.

% G-rated issues are non-trivial.

\[ E' = \text{H}^* \times (\frac{1}{2}; \mathbb{C} \times (\mathbb{Q}_2)) \]

if one extends to the finite version,

\[ E' = \text{H}^* (\frac{1}{2}; \mathbb{C} \times (\mathbb{Q}_2)) \]

special sequence defined by action gives a

H. Iwahori

Example $g = e^t$, $t \geq 0$, small, as before.

\[ \text{Then } \text{H}^* (\frac{1}{2}; \mathbb{C} \times (\mathbb{Q}_2)) = \text{H}^* (\mathbb{Q}) \text{ Eull.} \]
Goren and Segal: on $H^r(M)$.

Both approaches should receive careful attention.

For $c > g > -\varepsilon_0$ (small), as before,

standard axioms.

canonical: $\mathcal{D}$.

Advantage: no monodromy condition.

Write $H^r = \mathbb{C} \otimes \mathcal{D}_r$.

Consider the "fake" take.

Combine it.

$H^n(\prod \mathcal{D}_r)$

Equivariant product operation

$(\text{Cohen-Morley})$: there is an

Second approach (Tukey's, et al.)

$\exists \text{nullhomotopic } (\ast)$.

Then, can define a stable monodromy

that trivialization, $\mathcal{D} : H \to \mathcal{D}(r)$

vector bundle, and with respect to

In $\exists$ really nullhomotopic: $H$ as a symplectic

stable monodromy.

Type (Cohen--Jones--Segal): import

A first approach: there really monodromy

Second approach (Tukey, et al.)
Special properties (e.g., commutivity, associativity)

For somewhat different reasons, requires homotopy invariance (x).

Requiring homotopical invariance (x).

Multiplying by \( e_k \cdot e_0 \).

After multiplying map (denoted after localization map (denoted after)

Thus, we have (after cohomology) definition of a localization map \( e_0 \rightarrow \text{ht}(x) \rightarrow \text{ht} \).

Schematic squares:

\[
\begin{array}{c}
\text{ht} = e_0 \rightarrow \text{ht}(x) = e_0 \\
\text{ht} \times \text{ht} = e_0 \times e_0 \\
\text{ht} \times \text{ht} = e_0 \times e_0 \\
\end{array}
\]

Free homotopy type

Toric cohomology (based on earlier work of Fiedler, Gritsch, using logarithm

Relation? Hirsch's...
become isomorphic over $k((x))$.

\[ f(x) = \sum c_n x^n = c_0 + c_1 x + \cdots \]

or Euler class of the normal bundle. But

ospherical, then

\[ H^* (\mathbb{P}^2) \xrightarrow{\sim} H^*(\mathbb{P}^2) \]

\[ H^*(\mathbb{P}^2) \] is a smooth real algebraic.

Example (smooth, real algebraic)

\[ \dim H^*(\mathbb{P}^2) = \dim H^*(\mathbb{P}^2) \]

Corollary (smooth, real algebraic).

Second proof. Assume $M$ closed. We have

\[ H^2 (M, \mathbb{R}) = 0 \]}

in the formal version.

(b) in the tame version.

becomes an isomorphism and $\mathbb{R}^2(\mathbb{P}^2)$

\[ H^2 (\mathbb{P}^2) \xrightarrow{\sim} \mathbb{R}^2(\mathbb{P}^2) \]

Then the map induced by $\mathbb{R}^2(\mathbb{P}^2) \to \mathbb{R}$.

$M$ be a manifold with $\mathbb{R}$-action.

Local Torsion Theorem (80+) let
where $\kappa(x) = \text{Chern index (local invariant)}$.

Action terms:

$g \times : \kappa(x) \times \kappa(y) \rightarrow \kappa(x \times y)$

\[ H^g(\mathcal{Q}_{(g)}) \cong \text{Ht}_{(g)}(\pi) \]

Sketch of proof: Use action partitioning.

The homotopy invariant condition on $\pi$.

Homotopy invariance over $\mathbb{R}((u))$.

The $\mathbb{Q}$-module $\text{Ht}_{(g)}(\pi)$ is defined. This map becomes an isomorphism on $\mathbb{Q}$-homology.

The equivalence product

$\text{Ht}_{(g)}(\pi)$ The equivalence product

Remark: The cohomology of $(\pi)$ in a $\mathbb{Q}$-module of symplectic deformation.

Applications: To study classification.

Counter examples: Dim $\text{Ht}_{(g)}(\pi)$ $\subset$ Dim $H^g(\mathcal{Q}_{(g)})$.

Suppose the homotopic triviality condition is satisfied, so that the local action

$H^g(\mathcal{Q}_{(g)}) \cong \text{Ht}_{(g)}(\pi)$. 

Support the (Knudsen 2014).
Composition

derivation (degree n)

map in inverse

localization theorem:
on the "second product" of the classical operators on topological quantum field theory

H^X (y | CF_x (y)_2) \rightarrow H^X (c_2)

act by exchanging the ends

\frac{1}{2} \theta - rotation

H^X \rightarrow \text{order by}

version with symmetry:

H^X (\gamma_1 \otimes H^X (\gamma_2) \rightarrow H^X (\gamma_1) \otimes H^X (\gamma_2))
action of translation. Theorem 3: The translation action corresponds to a previous action of the Hecke algebra. This is because the translation action

corresponds to the previous action of the Hecke algebra by the


Hecke algebra.


Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):

Theorem 3.3 (2011):
Shcukat is already in the A-Hummaq-ParPN.

Slight deletion of different product  

Segre meta-comparison argument to (x)

counter by applying a specific

One would try to prove the

Hoped-for lemma should provide a TLP

It is a Calabi-Yau, work of Condo-

One can't expect this in general, but

\[ H^*(\mathbb{H}(\mathbb{Q}, \mathbb{Q})) \cong H^*(\mathbb{H}(\mathbb{A}, \mathbb{A})) \]

\[ (x) \]

\[ H^*(\mathbb{A}, \mathbb{A}) \cong \mathbb{A} \times \mathbb{A} \]

\[ \xrightarrow{\mathbb{A}} \]

\[ \mathbb{H}^*(\mathbb{A}, \mathbb{P}) \times \mathbb{C} \times \mathbb{A}, \mathbb{P} \]

have a product shcukat

More precisely, one would want to

General P, P is not really necessary.

Remark: the case \( P = \mathbb{A} \), which has

definition in the "universal case"

that it is sufficient to establish

Hence, they use these to show