ERRATUM TO "π_1 OF SYMPLECTIC AUTOMORPHISM GROUPS AND INVERTIBLES IN QUANTUM HOMOLOGY RINGS"

PAUL SEIDEL

ABSTRACT. We note an error in [2]. This Erratum will not be published.

The paper defines $Ham(M,\omega)$ to be the group of Hamiltonian automorphisms, equipped with the C^∞-topology, and G as "the group of smooth based loops in $Ham(M,\omega)$". This is a misleading formulation, since what the paper really means is that elements of G are Hamiltonian loops. If one understands it in that way, then the proof of [2, Lemma 2.1] as given is incorrect.

However, the distinction between "smooth loops in the symplectic automorphism group which remain inside $Ham(M,\omega)$" and "Hamiltonian loops" is ultimately irrelevant, because of the following:

Lemma. *Let $(\phi_t)_{0\leq t\leq 1}$ be a smooth isotopy of symplectic automorphisms of M, such that each ϕ_t is Hamiltonian. Then, the isotopy itself is a Hamiltonian isotopy.*

Proof. Let $a_t \in H^1(M;\mathbb{R})$ be the infinitesimal flux of the isotopy. This depends smoothly on t. If a_t is nonzero at some point $t \in (0,1)$, one can find arbitrarily small ϵ such that

$$\int_{t-\epsilon}^{t+\epsilon} a_t \, dt \neq 0. \tag{1}$$

By assumption, $\phi_{t-\epsilon}$ and $\phi_{t+\epsilon}$ are both Hamiltonian. By connecting them to the identity, one forms a loop in the symplectic automorphism group whose flux is (1). But this flux can be made arbitrarily small, contradicting [1]. Hence, a_t is necessarily identically zero. $\square$

After appealing to that, the proof of [2, Lemma 2.1] goes through as stated in the paper.

REFERENCES

[1] K. Ono. Floer-Novikov cohomology and the flux conjecture. *Geom. Funct. Anal.*, 16:981–1020, 2006.

[2] P. Seidel. π_1 of symplectic automorphism groups and invertibles in quantum homology rings. *Geom. Funct. Anal.*, 7:1046–1095, 1997.