December 11th - 18.02 - Fall 17

Worksheet: Center of mass, Moment of inertia

Center of mass. The first coordinate x_c of the center of mass of an object R of density δ is given by

$$x_c = \frac{1}{M}\iiint_R x\ \delta\ dV,$$

where $M = \iiint \delta\ dV$ is the total mass of R. The other coordinates of the center of mass are computed similarly.

Moment of inertia. The moment of inertia I_ℓ of an object R of density δ with respect to an axis ℓ is given by

$$I_\ell = \iiint_R d^2\ \delta\ dV,$$

where d is the distance to the axis ℓ.

Problem 1. Let C be a solid cone of half angle $\pi/4$ and height h, and let S be a conical shell with the same shape: S is the surface consisting of the slanted side of the cone C.

Compute the center of mass of C as well as its moment of inertia around its symmetry axis, assuming a constant mass denstiy δ_C so that its total mass $M_C = 1$.

Compare to the center of mass of S and to its moment of inertia around its symmetry axis respectively, assuming a constant mass denstiy $\delta_S =$ so that its total mass $M_S = 1$.

Problem 2.

Compute the moment of inertia of a plate of radius a and constant mass density δ rotating about:

(1) Its symmetry axis.
(2) An axis parallel to its symmetry axis, but passing through its edge.