Erratum to “A symmetric function generalization of the chromatic polynomial of a graph”

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October 2018

Theorem 3.4 is incorrect as stated because the definition of a maximal partition on p. 179 is misstated. The correct definition is that a partition \( \mu \vdash r \leq d \) with exactly \( l \) nonzero parts is maximal if \( \mu \) is allowable and for every other allowable partition \( \nu \) of an integer \( s \leq d \) (where \( s \) need not be equal to \( r \)), either \( \nu_i = \mu_i \) for all \( 1 \leq i \leq l \), or there exists \( i \leq l \) such that \( \nu_1 + \cdots + \nu_i < \mu_1 + \cdots + \mu_i \).

With this corrected definition, Theorem 3.4 and its corollaries are correct as stated.

The proof of Theorem 3.4 is essentially correct once the definition of “maximal” is corrected, but a few minor corrections need to be made. The definition of \( \varphi_\mu \) at the bottom of p. 179 should read

\[
\varphi_\mu(QS) = \begin{cases} 
1, & \text{if } r = d \text{ and } S = \{\mu_1, \mu_1 + \mu_2, \ldots, \mu_1 + \cdots + \mu_{l-1}\}, \\
t, & \text{if } r = d - 1 \text{ and } S = \{\mu_1, \mu_1 + \mu_2, \ldots, \mu_1 + \cdots + \mu_l\}, \\
t(t-1)^i, & \text{if } r \leq d - 2 \text{ and } S = \{\mu_1, \mu_1 + \mu_2, \ldots, \mu_1 + \cdots + \mu_i, \\
\mu_1 + \cdots + \mu_i + i + 1, \mu_1 + \cdots + \mu_i + i + 2, \ldots, d - 1\}, \\
0, & \text{otherwise},
\end{cases}
\]

and corresponding straightforward adjustments to the Proof of Claim on p. 180 should be made. Finally, on the third line from the bottom of p. 180, “\( u_\lambda \neq 0 \)” should be changed to “\( u_\nu \neq 0 \) for some \( \nu \geq \lambda' \)”, and “\( X_{P_\lambda} \)” in the displayed equation at the top of p. 181 should be changed to “\( X_{P_\lambda} \)”.

I am grateful to Timothy Chow for the preparation of this erratum.