ERRATA AND ADDENDA

 to

Enumerative Combinatorics, volume 1, second printing

by

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- p. 6, line 5–. Change compositon to composition.
- p. 11, Example 1.1.16, line 5. Change Y_i to Y_j (twice) and X_i to X_j .
- p. 16, line 3–. Change x_i to y_i .
- p. 19, line 19–. Change [k] to [n-k]. This correction needs to be made in the hardcover but not the paperback edition.
- p. 19, line 4–. This should say $0 \le a_i \le x + n i 1$. It is correct in the hardcover edition and incorrect in the paperback edition.
- p. 19, line 7–. Change sufficies to suffices.
- p. 20, line 7–. It would be more accurate to replace "The proof of Proposition 1.3.7" with "The third proof of Proposition 1.3.4".
- p. 24, Proposition 1.3.14, part 3, line 2. Replace "k vertices" with "k-1 vertices". One does not need the bijection $\pi \to T(\pi)$ to see this. Any binary tree with k endpoints has k-1 vertices with two successors.
- p. 27, line 7. Add "let" after "Now".
- pp. 30 and 31, Figures 1-6 and 1-7. The shading of these figures that appeared in the original printing was omitted from the second printing. In Figure 1-6, the boxes are shaded to denote the Young diagrams of the partitions Ø, (1), (2), (1, 1), (3), (2, 1), (3, 1), (2, 2), (3, 2), (3, 3) in that order. In Figure 1-7, the boxes should be shaded above the lattice path L so that the shaded boxes form the Young diagram of the partition (4, 3, 1).
- p. 33, Twelvefold Way, entries 7 and 10. The sum for entry 7 should begin S(n, 0). Similarly the sum for entry 10 should begin $p_0(n)$. These terms are only relevant when n = 0 and yield the correct values $S(0, 0) = p_0(0) = 1$.
- p. 34, line 9–. Change (24a) to (24b).

 p. 42, lines 2–3. Further surveys of estimating the solution to an enumeration problem are A. M. Odlyzko, in *Handbook of Combinatorics*, vol. 1, Elsevier, Amsterdam, 1995, pp. 1063–1069, and the book P. Flajolet and R. Sedgewick, *Analytic Combinatorics*, to appear. Excerpts from this latter book are available at

http://algo.inria.fr/flajolet/Publications/books.html

- p. 45, Exercise 8(b). Add at the end of this exercise: "(Set $\binom{m}{i} = 0$ if i < 0.)"
- p. 47, Exercise 19(c), line 5. Change f(n) to $f_k(n)$.
- p. 50, Exercise 1.40. The statement of this exercise is somewhat misleading, since the solution gives a formula for a_i not in terms of the f_n 's, but rather in terms of the g_n 's defined by $\log F(x) = \sum_{n>1} g_n x^n$.
- p. 52, Exercise 2(a), line 3. Change x + i + n + 1 to x + n + 1.
- p. 55, Exercise 9(b). A simple combinatorial proof was given by the Cambridge Combinatorics and Coffee Club (December 1999).
- p. 56, Exercise 13, line 2. Change $b_1 < b_2 < \cdots < b_m$ to b_1, b_2, \cdots, b_m .
- p. 56, Figure 1-14. The next-to-last dot should be circled.
- p. 59, Exercise 26, line 3. Change $m_k(\mu)$ to $f_k(\mu)$.
- p. 59, Exercise 26, line 2–. Change function to functions.
- p. 59, Exercise 26. The following historical remarks concerning this exercise may be of interest. I discovered the result in 1972 and submitted it to the Problems and Solutions section of the Amer. Math. Monthly. It was rejected with the comment "A bit on the easy side, and using only a standard argument." My guess is that the editors did not understand the actual statement and solution of the problem. I had mentioned the result to Daniel I. A. Cohen, who included the case k = 1 as Problem 75 of Chapter 3 in his book Basic Techniques of Combinatorial Theory, Wiley, New York, 1978. For this reason the case k = 1 is sometimes called "Stanley's theorem." An independent proof of the general case was given by Kirdar and Skyrme, as mentioned in the text (page 59). The generalization from k = 1 to arbitrary k was independently found by Paul Elder in 1984, as reported by R. Honsberger, Mathematical Gems III, Mathematical Association of America, 1985 (page 8). For this reason the general case is sometimes called "Elder's theorem." A further proof was given by A. H. M. Hoare, Amer. Math. Monthly **93** (1986), 475–476.
- p. 61, Exercise 33, line 1. Change $A(n,k)2^k$ to $A(n,k+1)2^k$, and change k at the end of the line to k-1.

- p. 62, line 8–. Change $\sum_{n\geq 0}$ to $\sum_{i\geq 0}$.
- p. 62, line 1–. Change \mathfrak{S}_{2n+1} to \mathfrak{S}_{2n-1} .
- p. 63, Exercise 45. A plausible explanation of the number 103,049 was found by David Hough and is discussed in R. Stanley, *Amer. Math. Monthly* 104 (1997), 344–350. A less convincing explanation of the number 310,952 appears in L. Habsieger, M. Kazarian, and S. Lando, *Amer. Math. Monthly* 105 (1998), 446. See also page 213 of *Enumerative Combinatorics*, vol. 2.
- p. 67, line 3. Insert a space before "has".
- p. 70, equation (21). Change s_j to s_i (twice).
- p. 71, line 12–. Add the following sentence before this line (which is needed in the statement of Theorem 2.4.1).

Define the rook polynomial $r_B(x)$ of the board B by

$$r_B(x) = \sum_k r_k x^k.$$

- p. 72, line 10. Change "If" to "It".
- p. 75, Theorem 2.4.4, lines 4–5. Change "only $s_1 \ge 0$ (i.e., $s_i < 0$ for $2 \le i \le t$ " to $s_1 = 0$ and $s_i < 0$ for $2 \le i \le t$ ".
- p. 80, lines 14– and 16–. Change "positive" to "nonnegative".
- p. 80, lines 7– and 11–. Change τ' to $\tilde{\tau}$.
- p. 81, Figure 2-1. Change τ' to $\tilde{\tau}$.
- p. 82, §2.7, line 7. Change v_{i+i} to v_{i+1} .
- p. 84, lines 11–13. Change the sentence "Property (a) ... obtained from L." to "Property (a) follows since the triple (i, j, v) can be obtained from L* by the same rule as it can be obtained from L."
- p. 84, Example 2.7.2, line 7. Change α_i and δ_i to α_j and δ_j (twice).
- p. 85, Notes. Ferrers boards were first considered by D. Foata and M. P. Schützenberger, On the rook polynomials of Ferrers relations, *Colloquia Mathematica Societatis Janos Bolyai*, 4, Combinatorial Theory and Its Applications, vol. 2, (P. Erdős, A. Renyi, and V. Sós, eds.), North-Holland, Amsterdam, 1970, pp. 413–436.
- p. 88, equation (44). Change $1/n^5$ to $2/n^5$.

- p. 89, Exercise 11, line 3. Change "nents" to "nent".
- p. 89, Exercise 11(b), line 5. Change G to \overline{G} .
- p. 92, line 5. Change A_T^a to V_T^a .
- p. 92, items **e,f**. Change |T| to $|A_T|$.
- p. 93, Exercise 8(d). Change f(n) + f(n+1) to $f_2(n) + f_2(n+1)$.
- p. 94, solution to Exercise 14, line 5. Change $k \leq -1$ to $k \geq -1$.
- p. 95, line 2. Change "regular" to "that every connected component is regular".
- p. 96, line 3 after equation (1). Delete comma after C.
- p. 97, Example 3.1.1(d), line 3. Change [9] to 9.
- p. 111, Example 3.5.3, line 6. Change second bacde to badce.
- p. 111, Figure 3-24. The shading of this figure that appeared in the original printing was omitted from the second printing. The entire inside region should be shaded.
- p. 111, line 3-. Change "m element" to "m-element".
- p. 112, lines 7- to 6-. Change $e(J(\boldsymbol{m}+\boldsymbol{n})) = e(\boldsymbol{m}+\boldsymbol{1}\times\boldsymbol{n}+\boldsymbol{1}) = \binom{m+n}{n}$ to $e(\boldsymbol{m}+\boldsymbol{n}) = \binom{m+n}{n}$.
- p. 117, line 13. Change T < 1 to $T < \hat{1}$.
- p. 117, line 4–. After \Leftrightarrow insert "f(0) = g(0) and".
- p. 120, line 10. Insert Δ after "collection".
- p. 122, Figure 3-29. Shading is missing from Γ_4 , Γ_5 , and Γ_6 . All two-dimensional regions (including the outside one) are 2-cells.
- p. 125, line 6. Change \mathbb{C} to K.
- p. 125, line 9–. Change \mathbb{C} to K.
- p. 125, line 8–. Change $y \leq 1$ to $y \leq \hat{1}$.
- p. 127, lines 2 and 6. Change L to L_n .
- p. 127, Example 3.10.3, line 2. Change "Note" to "For the purpose of this example, we say". If one wants to retain the more standard convention that the empty set spans $\{0\}$, then we need to enlarge $L_n(q)$ by adding \emptyset below $\{0\}$.

- p. 128, line 14. The type of a set partition $\pi \in \Pi_n$ has not been defined, though the definition should be clear in analogy to the type of a permutation. Namely, define $\operatorname{type}(\pi) = (a_1, \ldots, a_n)$ if π has a_i blocks of size i.
- p. 128, line 16. Change = to \cong .
- p. 131, line 6 of text. Change "rank i" to "of rank i".
- p. 131, line 5–. Change $\sigma: P \to [n]$ to $\sigma: P \to \mathbf{n}$.
- p. 133, line 13–. Change $a_0 = \hat{0}, a_{s+1} = \hat{1}$ to $a_0 = 0, a_{s+1} = n$.
- p. 136, line 6. Change $(-1)^n \Delta Z(P, m-1)$ to $(-1)^{n-1} \Delta Z(P, m-1)$.
- p. 137, line 7. Change Q to P Q.
- p. 139, Exercise 1.193, line 2. Change γ_n to $\gamma(n)$.
- p. 142, line 14–. It should have been stated that card(x, y) is short for card([x, y]).
- p. 152, reference 12, line 2. Change *functions* to *function*.
- p. 153, Exercise 1a, line 1. Change "operation" to "relation".
- p. 155, line 4. Insert "irreducible" before "*connected*". (An irreducible poset is one that cannot be written in a nontrivial way as a direct product.)
- p. 156, Exercise 15(d). This exercise was solved by J. D. Farley and R. Klippenstine, Posets with the same number of order ideals of each cardinality, II, preprint dated November 30, 2004.
- p. 157, Exercise 22(e). It was shown by J. Farley, J. Combinatorial Theory (A) 90 (2000), 123–147, that the only nondecreasing cover functions (with $f(0) \ge 1$) are f(n) = k and f(n) = n + k for $k \ge 1$. This confirms a conjecture of R. Stanley, Fibonacci Quart. 13 (1975), 215–232 (page 226).
- p. 160, line 1. Change second P to \overline{P} .
- p. 160, Exercise 32, line 4. Insert $\mu(x_k, \hat{1})$ after $\mu(x_{k-1}, x_k)$.
- p. 164, line 1. Change a_i to y (twice).
- p. 166, line 1–. Change X_1 to H_1 and X_{ν} to H_{ν} .
- p. 167, Exercise 59a. Change the last sentence to: Show that if $Z(P, m + 1) = \sum_{i\geq 1} a_i \binom{m-1}{i}$, then $Z(Q_0, m + 1) = 1 + \sum_{i\geq 1} a_i m^i$.
- p. 170, Exercise 70(a), line 2. Change $\beta(P, S)$ to $\beta(P_n, S)$.

- p. 174, Exercise 81a, lines 3–4. Change "define f, g" to "define g, h".
- p. 174, Exercise 81c, line 5. Change this line to

$$1 + t \sum_{n \ge 1} G_n(q, t) x^n / (n)! = \left[1 - t \sum_{n \ge 1} (1 - t)^{n-1} x^n / (n)! \right]^{-1}.$$

- p. 174, Exercise 81c, line 2–. Change $(1-t)/(e^{x(t-1)}-1)$ to $(1-t)/(e^{x(t-1)}-t)$.
- p. 175, Exercise 7(b). Replace the first paragraph with the following: Suppose that $f: \operatorname{Int}(P) \to \operatorname{Int}(Q)$ is an isomorphism. Let $f([\hat{0}, \hat{0}]) = [s, s]$, where $\hat{0} \in P$ and $s \in Q$. Define A to be the subposet of Q of all elements $t \geq s$, and define B to be all elements $t \leq s$. Check that $P \cong A \times B^*$, $Q \cong A \times B$.
- p. 178, solution to Exercise 19a, lines 5–7. Interchange f_k and g_k .
- p. 183, solution to Exercise 30, line 5. Change Q to \overline{P} (under the summation sign).
- p. 184, solution to Exercise 32, line 3. Insert $\mu(x_k, \hat{1})$ after $\mu(x_{k-1}, x_k)$.
- p. 184, solution to Exercise 37b, line 1. Change "f(x,s) = x" to " $f(x,s) = \phi(x)$ (so F(x,s) = x)".
- p. 187, line 2. Remove $\prod_{i=1}^{k}$.
- p. 187, line 3-. Change $x^{n-\dim W}$ to $x^{n-\dim W'}$.
- p. 188, Exercise 46, second solution, lines 1–2. Change $a \neq \{1\}$ to $\emptyset \neq a$.
- p. 190, line 1. Change $x_1^{a_1\cdots}x_n^{a_n}$ to $x_1^{a_1}\cdots x_n^{a_n}$.
- p. 191, Exercise 50, line 4. Under the third summation symbol, change $x \in P_j$ to $y \in P_j$.
- p. 191, Exercise 51, line 5. Change "voltage graphs" to "gain graphs".
- p. 191, Exercise 51, line 7. A general reference for enumerative results on gain graphs is T. Zaslavsky, J. Combin. Theory Ser. B 64 (1995), 17–88.
- page 192, line 2–. This formula should be:

$$Z(P \oplus Q, m) = Z(P, m) + Z(Q, m) + \sum_{j=2}^{m-1} Z(P, j) Z(Q, m+1-j), \quad m \ge 2$$

• p. 193, line 1. Change "chains" to "multichains".

- p. 195, solution to Exercise 61b. Interchange p and p1 throughout.
- p. 196, line 16–. Change $x_{\rho(x_i)}$ to $t_{\rho(x_i)}$.
- p. 197, Exercise 69(d), lines 2– to 1–. Update this reference to *Discrete Math.* **79** (1990), 235–249.
- p. 197, Exercise 70(a). Change $\beta(P, S)$ to $\beta(P_n, S)$.
- p. 200, Exercise 80, line 4. Change 6n+3 to 6n+3, i.e., the 3 should not be italicized. Could this be the most nitpicking error of this errata?
- p. 206, first line of proof. Change R(x) to F(x).
- p. 216, line 5. Change 1.3.3 to 1.3.
- p. 223, line 17. Change P to C.
- p. 224, line 3. Change "rank $d = \dim \mathcal{C}$ to "rank d + 1 where $d = \dim \mathcal{C}$ ".
- p. 227, line 8. Change " $a_i = \lceil b_i \rceil$, the least integer $\geq b_i$ " to " $a_i = \lceil b_i 1 \rceil$, where $\lceil b_i \rceil$ denotes the least integer $\geq b_i$ ".
- p. 230, line 2 of Proof. Change $d \dim \sigma$ to $d \dim \sigma + 1$.
- p. 231, line 3. Change the initial minus sign to +.
- p. 231, Lemma 4.6.17(i). As stated, the result is false. For instance, let E = N and a₁ = -1. Then G(λ) = 1 but E(λ⁻¹) = ∑_{n≥0} λ⁻ⁿ. We need to assume also that g(r) > 0 for at least one r > 0. We claim that then g(s) = 0 for all s < 0, and hence (i) follows. Let α ∈ E satisfy L(α) = r > 0, and suppose that there exists β ∈ E with L(β) = s < 0. Then for all t ∈ N the vectors -tsα + trβ are distinct elements of E, contradicting g(0) < ∞.
- p. 236, line 6. Change "union" to "intersection".
- p. 232, line 6 (excluding the heading). Change "matrix" to "N-matrix".
- p. 233, line 9. insert $n \times n$ before \mathbb{P} -matrices.
- p. 233, lines 6– and 5–. A subscript n is missing from H four times.
- p. 235, lines 2– and 1–. Change \mathcal{P} to \mathcal{P}_m .
- p. 237, line 8. Change den (γ, t) to den (γ/t) .
- p. 240, Example 4.6.32(b), line 2. Change the coefficient of $\overline{i}(\mathcal{P}, 1)$ from -1 to 1.

- p. 241, line 6. Change a_k to a_p .
- p. 244, Figure 4-13. Add an edge from 13 to 31.
- p. 246, lines 5–6. Change 7 to 6 (twice).
- p. 249, line 15–. Add at end of line: where $b(n) = \sum_{v \in \mathcal{B}_n} w(v)$.
- p. 253, line 11–. Change i-th to kth.
- p. 253, line 4–. Change D_4 to D_3 .
- p. 256, line 2. Change (j 1, j) to (j, j 1).
- p. 256, line 10. Change to +.
- p. 257, line 19–. Change $f_s(n_s)$ to $f_k(n_k)$.
- p. 257, line 18–. Change $f_s(n_{s+1})$ to $f_k(n_{k+1})$.
- p. 257, line 8–. Change $f_{i_j}(n_{j+1})$ to $f_{i_j}(n_j)$.
- p. 260, Figure 4-42. Change the label on the edge from 00 to 01 from $F_1 * F_2$ to $F_1 * F_3$.
- p. 262, line 5-. The first published statement for the generating function for F(x) appearing before equation (47) seems to be due to H. N. V. Temperley, *Phys. Rev. (2)* 103 (1956), 1–16.
- p. 262, line 2–. The result of Hickerson has now appeared in *J. Integer Sequences* (electronic) **2** (1999), Article 99.1.8,

http://www.research.att.com/~njas/sequences/JIS/HICK2/chcp.html.

- p. 265, line 2–. Change $\mathbb{Z}/n\mathbb{Z}$ to $(\mathbb{Z}/n\mathbb{Z})^2$.
- p. 266, Exercise 12. Change $\Phi \alpha = 0$ to $\Phi \alpha = \beta$, where β is a vector of linear polynomials an + b. Moreover, the final sentence should be "Show that \mathbb{N}^m can be partitioned into finitely many regions such that on each region the number of solutions is a quasipolynomial in n for n sufficiently large." It is possible that this result is already known.
- p. 271, Exercise 27(a), line 2. Change this line to

$$x_1 + x_2 + \dots + x_r \le 1, \ y_1 + y_2 + \dots + y_s \le 1, \ x_i \ge 0, \ y_i \ge 0.$$

• p. 271, Exercise 28(d). We must also define $i(\mathcal{Q}, 0) = 1$, despite the fact that the value of the polynomial $i(\mathcal{Q}, n)$ at n = 0 is $\chi(\mathcal{Q})$, the Euler characteristic of \mathcal{Q} .

- p. 277. Exercise 5, line 2. Change $F(0) \neq 0$ to $G(0) \neq 0, \infty$.
- p. 280, Exercise 14(a), first line of ii. Change V_i to v_i .
- p. 281, line 11. Change x^y to x^{γ} .
- p. 285, Exercise 23. Peter McNamara has pointed out a gap in the proof. Namely, from the fact that P - M and P - M' are disjoint unions of chains, it need not follow (when P has maximal chains of length one) that P is a disjoint union of chains, together with the stated relations x < y. To fix the proof, note that m is the largest power of x_0 that can appear in a monomial in $\overline{G}(P, \mathbf{x})$. Hence m is the largest power of any x_i that can appear in a monomial in $\overline{G}(P, \mathbf{x})$. Let A be an antichain of P. We can easily find a strict P-partition that is constant on A, so $\#A \leq m$. Hence the largest antichain of P has size m. By Dilworth's theorem, P is a union of m chains. Each such chain intersects M and M'. It is now easy to see that if P - M and P - M' are disjoint unions of chains, then indeed P is a disjoint union of chains together with the stated relations x < y, and the proof proceeds as before.
- p. 290, Exercise 34(a), line 1. Change $(1, \zeta, \zeta^{2r}, \dots, \zeta^{(n-1)r})^t$ to $(1, \zeta^r, \zeta^{2r}, \dots, \zeta^{(n-1)r})^t$.
- p. 293, line 9–. In the definition of a connected graph, it should also be specified that the empty graph is *not* connected.
- p. 294, line 5. Change "define" to "defining".
- p. 294, line 8. Insert a comma after $\{1, 3, 4, 5, 8, 9\}$.
- p. 294, line 8–. Change "defined" to "define".
- p. 295, Figure A-2. Interchange the labels 3 and 5 on the third tree.
- p. 308, Problem 7, line 3. Change [n] to [n-1].
- p. 310, Problem 25, line 2. Change "occurrences" to "occurrences" (twice).
- p. 310, Problem 25, line 3. Change "t occurrences of each a_{ij} " to "2t occurrences of each a_{ij} ".
- p. 314, Problem 6(c). A solution was found by Ethan Fenn (private communication, November, 2002). The rating should be changed to [3–] and the problem restated as follows.

Let $k \geq 3$, and let P_k denote the poset of all subsets of \mathbb{P} whose elements have sum divisible by k. Given $T \leq S$ in P_k , let

$$i_j = \#\{n \in T - S : n \equiv j \pmod{k}\}.$$

Clearly $\mu(S,T)$ depends only on the k-tuple $(i_0, i_1, \ldots, i_{k-1})$, so write $\mu(i_0, \ldots, i_{k-1})$ for $\mu(S,T)$. Show that

$$\sum_{i_0,\dots,i_{k-1}\geq 0} \mu(i_0,\dots,i_{k-1}) \frac{x_0^{i_0}\cdots x_{k-1}^{i_{k-1}}}{i_0!\cdots i_{k-1}!}$$
$$= k \left[\sum_{j=0}^{k-1} \exp\left(x_0 + \zeta^j x_1 + \zeta^{2j} x_2 + \dots + \zeta^{(k-1)j} x_{k-1}\right) \right]^{-1},$$

where ζ is a primitive kth root of unity.

- p. 310, Problem 28, line 4. Change L_{n+1} to L_n .
- p. 314, Problem 10, line 1. Delete the difficulty rating [2+] at the beginning of the line.
- p. 314, Problem 13, lines 2–3. Delete the difficulty rating [2+] at the beginning of these lines.
- p. 314, line 2–. Delete "(ii)".
- p. 318, Problem 10, line 3. Change $H(\boldsymbol{a})$ to $H_n(\boldsymbol{a})$.
- p. 318, Problem 11(b), line 1. Insert "the" after "with".
- p. 318, Problem 12(b), line 2. Change $\sum_{\alpha \in \Phi_3}$ to $\sum_{\alpha \in E_{\Phi_3}}$.
- p. 318, Problem 13. This should be rated [2+].
- p. 319, line 9–. This erratum is unnecessary and can be deleted.
- p. 320, line 2 (paperback edition only). Change $(2^{a,1} 1) \cdots (2^{a,1} 1)$ to $(2^{a_1-1} 1) \cdots (2^{a_k-1} 1)$.
- p. 320, lines 16–17. Delete this item.
- p. 321, item 3, line 2. Change $\mu(kn)$ to $\mu_S(kn)$.
- p. 321, item 6. The computation of f(14) = 1338193159771 is given by J. Heitzig and J. Reinhold, Order 17 (2000), 333–341. They also compute the number of labelled *n*-element posets for $n \leq 16$. According to Vledeta Jovović, as reported on the Encylopedia of Integer Sequences, A000112, http://www.research.att.com/~njas/sequences, we have f(15) = 68275077901156 and f(16) = 4483130665195087.
- p. 321, line 7. Delete this entry (for p. 149, line 10).

- p. 322, lines 8- to 5-. The stated result is false. The hypothesis on L should be the following: L is an n-element lattice such that for all 0 < x ≤ y in L, there exists z ≠ y such that z ∨ x = y.
- p. 322, line 4–. Change ℓ^{k-1} to $k^{\ell-1}$ $(\ell \ge 2)$.
- p. 324, line 9–. Change **22** to **29**.