

ERRATA

for *Enumerative Combinatorics*, vol. 1, 2nd ed., 2012

version 28 November 2024

- page 5, line 9–. Insert “a” after “is”.
- page 23, line 6. Insert “a” after “is”.
- page 24, line 2. Change c_i to c_1 .
- page 34, line 15. It should be stated that for this second way, $d + 1$ cannot be inserted at the end even though $d \notin D(w)$.
- page 38, line 17. Change “such letter appears in exactly one compartment” to “compartment contains exactly one such letter”.
- page 38, line 18. Change “does” to “do”.
- page 40, line 15. The intent of the parenthetical comment was to write the summation as

$$\sum_{g(4) \geq g(6)} \sum_{g(6) \geq g(3)} \dots \sum_{g(5) \geq 0} q^{g(4) + \dots + g(5)}.$$

We are actually first summing on $g(5) \geq 0$, then $g(1) \geq g(5)$, etc.

- page 40, line 4–. Insert $\sum_{m \geq 0}$ before $\sum_{w \in \mathfrak{S}_d}$.
- page 43, line 4. Change “relection” to “reflection”.
- page 43, line 14. Change $i \geq 0$ to $i \geq 1$.
- page 45, line 1. Change w to $T(w)$.
- page 46, line 13–. Remove bracket at end of line.
- page 47, line 11. Change $n \geq 2$ to $n \geq 1$.
- page 49, lines 20–22. The two sentence beginning “Figure 1.10 ...” and “Let $f(n)$ denote ...” should be interchanged, since $f(n)$ is used in the first of these sentences but defined in the second.

- page 50 , line 4. Change “tree” to “trees”.
- page 53, line 1-. Change n to $n - 1$.
- page 54, line 8. Change “number” to “numbers”.
- page 61, Figure 1.15. Change $w = 1212112$ to $w = 12121121$.
- page 62, equation (1.78). Change $j \in \mathcal{S}_i$ to $j \in S_i$.
- page 62, line 13-. Change \mathcal{S}_j to S_i .
- page 65, line 3. Perhaps it is misleading to say that the bijection is “easy to check”.
- page 65, line 7. Change proposition to Proposition.
- page 66, line 13. Change $F(0, q)$ to $F(x, 0)$.
- page 68, line 17-. Change “ k of the γ_i ’s” to “ k γ_i ’s”.
- page 70, lines 3 and 5. Replace “replacing” with “placing”.
- page 71, line 2-. Change “then then” to “then the”.
- page 76, line 2. Change “ x to $-x$ ” to “ t to $-x$ ”.
- page 77, line 5-. Change “ $f(0), f(0)$ ” to “ $f(0), f(1)$ ”.
- page 78, line 5. Note that in the formula $\Delta(x)_n = n(x)_{n-1}$, Δ is acting on x , not on n as before.
- page 85, Theorem 1.10.4, line 3. Change $\text{GL}(n, q)$ to $\text{GL}(m, q)$.
- page 88, line 11-. Change m_k to $m_k - 1$.
- page 88, line 13-. Change v_{ij} to v_{1j} .
- page 89, line 3-. Change 1.8.6(c) to 1.8.6(a).
- page 92, line 5. Change $f \neq 0$ to $f \neq z$.
- page 92, line 7. Insert the exponent -1 at the end of this line.
- page 94, line 15. Change $\psi(F)$ to $\psi(A)$.

- Lemma 1.10.14, line 3. The case $n = 0$ is unnecessary since the formula for a_n is valid for $n = 0$.
- Proof of Lemma 1.10.14. Bruce Sagan notes that there is a simple combinatorial proof that $a_n = (q - 1)^{n-1} - a_{n-1}$. The proof of the lemma follows by induction.
- page 96, line 3-. Change $\#\Gamma_{\text{id}}$ to $\#(\text{GL}_0(n, q) \cap \Gamma_{\text{id}})$.
- page 98, line 7. Change “in the monograph” to “is the monograph”.
- page 98, line 20. Change Rodriques to Rodrigues.
- page 98, line 16-. Change “papers [1.23][1.24]” to “paper [1.24]”.
- page 100, line 15-; page 101, line 1; page 101, item [14]; page 589, entry “Buckheister”. Change “Buckheister” to “Buckhiester”.
- page 104, Exercise 3.3(e). We should assume $n \geq 1$.
- page 109, Exercise 35(b,c,d). For the history of these results, see MathOverflow 63561.
- page 111, line 2. Change “second” to “third”.
- page 111, Exercise 49, line 1. Change (1.38) to (1.37).
- page 112, Exercise 54(a), line 1. Change $A(2n+1, n+1)$ to $A(2n+1, n)$.
- page 112, Exercise 55. One should compute $\text{maj}(w^r)$ in terms of both $\text{maj}(w)$ and $\text{des}(w)$, not just $\text{maj}(w)$.
- page 112, Exercise 55, line 2. Delete “, respectively”, since $\text{maj}(w^r)$ is expressed in terms of both $\text{des}(w)$ and $\text{maj}(w)$.
- page 113, Exercise 1.58(a), line 2. Change “and” to “can”.
- page 114, Exercise 61(b). It would be better to use a different letter than u , since u has different meaning in part (a).
- page 114, Exercise 61(c). Although there is technically no error, the terminology in parts (a) and (c) is inconsistent. In (a) the peaks and valleys are the terms w_i , while in (c) the double falls are the indices i .

- page 114, Exercise 62(b), line 2. Change k to i (twice) (in order to be consistent with the notation of the solution on page 160).
- page 115, Exercise 70(b), line 3. Change $(1, 0)$ to $(0, 1)$, and change “i.e.” to “i.e.,”.
- page 120, Exercise 94, second bullet. Change “Then” to “The”.
- page 121, Exercise 96(c), line 3. Change 4.4.1.1 to 4.1.1.
- page 121, Exercise 98. Strictly speaking, when we put $q = \zeta$ in the definition of $\begin{bmatrix} n \\ k \end{bmatrix}$ on page 55, we get $0/0$. To be precise we should write $\lim_{q \rightarrow \zeta} \begin{bmatrix} n \\ k \end{bmatrix}$.
- page 123, Exercise 116(b), line 2. Change distribuiton to distribution.
- page 125, line 3. Change $\Delta^j g_k(-n)$ to $\Delta^j g_k(-n)|_{n=0}$.
- page 126, line 2. Change $1 \leq i < j \leq n$ to “ $1 \leq i \leq j \leq n$, excluding $(i, j) = (1, n)$,”.
- page 126, Exercise 128(d), line 7. Change $\frac{2}{1+x}$ to $\frac{2x}{1+x}$.
- page 126, Exercise 129(a), line 3. Add $\sum_{k \geq 0}$ after $\sum_{n \geq 1}$.
- page 127, Exercise 135, line 2. Change “ n even” to “ n even, $n \neq 0$ ”.
- page 132, Exercise 161(a), lines 2 and 3. Change $\Phi_{k,0}$ to $\Psi_{k,0}$ (in order to be consistent with the previous exercise and to avoid possible confusion with the cyclotomic polynomial $\Phi_k(x)$). It would also be better notationally to write $F_k(x)$ instead of $\Psi_{k,0}(x)$ on line 3.
- page 136, line 1–. Change q^{n-1} to q^{k-1} .
- page 137, Exercise 184(a), line 5. Change “polynomials $f(x) = \beta$ for $0 \neq \beta \in \mathbb{F}_q$ are” to “polynomial $f(x) = 1$ is”.
- page 137, Exercise 184(b), line 1. To be consistent with (a), change $f(n, q)$ to $f_q(n)$.
- page 138, Exercise 186(a), line 3. Insert “of Exercise 1.185” after “(b)”.
- page 138, equation (1.148). Change q^{m+n-1} to $q^{m+n-1}(q-1)$.

- page 139, Exercise 193, line 2. Change γ_n to $\gamma(n)$.
- page 140, Exercise 199, line 4. The condition (only relevant in characteristic 2) should be that $A = -A^t$ and A has 0's on the main diagonal. Some people will take this to be the definition of “skew-symmetric,” while others will call these matrices “alternating.”
- page 140, Exercise 198(b), last displayed equation. Change (q^{2m-1}) to $(q^{2m-1} - 1)$ (twice).
- page 148, Exercise 56, line 3. Change $=$ to \subseteq .
- page 150, Exercise 22(b), line 6. Change x^{2i+1} to x^{2i-1} .
- page 151, Exercise 25, line 3-. Change “ r of these parts” to “ s of these parts”.
- page 151, Exercise 25, line 3-. Change $\binom{r+s}{r}$ to $\binom{r+s}{s}$. Of course these two binomial coefficients are equal, but it is more logical to use $\binom{r+s}{s}$.
- page 152, Exercise 35(f), lines 7–8. In the sentence “Replace ...”, change the two 1's to 2's, and change the 2 to 1.
- page 152, Exercise 35(f), line 9. Change each 1 to 2 and each 2 to 1.
- page 153, Exercise 36, line 2. Change $f_k(1)$ to $f_1(k)$ (twice).
- page 155, Exercise 43, line 8-. Change j to $n - j$.
- page 156, line 2. Change *Combinatorica* to *Combinatoria*.
- page 156, Exercise 49, line 3. Change the comma at the end of the line to a period.
- page 156, Exercise 49, line 4. Delete this line.
- page 156, Exercise 49, line 5. Delete **b.** at the beginning of the line.
- page 156, line 9-. Change $f(x) = 0$ to $(1 - x)^{-d-1}A_d(x) = 0$.
- page 159, line 8-. Change $F - bx$ to $F - x$.
- page 160, line 12. Change $g(n + 1) +$ to $g(n + 1) =$.

- page 160, Exercise 62. Change **a.**, **b.**, **c.** to **b.**, **c.**, **d.**.
- page 160, lines 5–6. Perhaps the sentence “This bijection . . . and so on.” would be clearer if it were replaced with “*Hint.* This bijection has the property that $\max\{a_1, \dots, a_n\} = \text{des}(\rho(w)^{-1}) + 1$.”
- page 163, Exercise 80. For the convoluted history of this result, see R. Gilbert, A Fine rediscovery, *Amer. Math. Monthly* **122** (2015), 322–331. In particular, the result is actually first due to N. J. Fine in 1959.
- page 164, Exercise 83, lines 7–8. Change (a, a) to (u, u) and $(a + 1, a)$ to $(u + 1, u)$ (since a has already been used as an indeterminate).
- page 164, Exercise 83, line 4–. Change the second denominator factor $(1 - a^j b^j c^{j-1} d^{j-1})$ to $(1 - a^j b^{j-1} c^j d^{j-1})$.
- page 165, Exercise 89, line 7. Change n to k (twice).
- page 166, line 1. While this sentence is correct, there is direct agreement with equation (1.88) if one substitutes $-q^{-1/2}$ rather than $-q^{1/2}$ for x .
- page 166, Exercise 91(d), line 4. Replace this line with

$$1 + \sum_{n \geq 1} (-1)^n (x^n - x^{-n-1}) q^{\binom{n}{2}}.$$

- page 166, Exercise 92. This identity is actually due to I. Gessel and D. Stanton, *Trans. Amer. Math. Soc.* **277** (1983), 173–201 (equations (7.13) and (7.15)).
- page 168, Exercise 104, line 1. Notation is confusing! Change $f(x)$ to $F(x)$.
- page 168, Exercise 104, line 1. Change $i^2 = 1$ to $i^2 = -1$ and $f(x)$ to $F(x)$.
- page 168, Exercise 104, line 2. Change the last four f 's to F .

- page 168, Exercise 104, line 4. Change the four lines with a left brace to

$$= \begin{cases} \frac{1}{4} (10^n + (-1)^k 2^{2k+1}), & n = 4k \\ \frac{1}{4} (10^n + (-1)^k 2^{2k+1}), & n = 4k + 1 \\ \frac{1}{4} 10^n, & n = 4k + 2 \\ \frac{1}{4} (10^n - (-1)^k 2^{2k+2}), & n = 4k + 3. \end{cases}$$

- page 169, Exercise 108, line 3-. Change “in papers” to “in the papers”.
- page 169, Exercise 109(a), line 4. Change “is bijection” to “is a bijection”.
- page 169, Exercise 111. The first sentence may be unclear. We are counting the number of ways to choose a k -element subset S of $[n]$ and then partition S into j intervals.
- page 170, Exercise 117. A simpler proof was suggested by Bruce Sagan. Let the cycle containing 1 be $(1, a_2, \dots, a_k)$. There are $(n - 1)(n - 2) \cdots (n - k + 1)$ ways to choose a_2, \dots, a_k and then $(n - k)!$ ways to permute the elements not in this cycle, etc. Perhaps the difficulty rating of this exercise should be [2].
- page 170, Exercise 118(b). As pointed out by Bruce Sagan, there is a simple solution analogous to the solution above for Exercise 117.
- page 171, line 4. The index entries for these four names is missing.
- page 171, line 6. Insert a space between “Let” and v .
- page 172, line 6. Change $(-1)^j b_{k-j}$ to $(-1)^j \binom{-n}{j} b_{k-j}$.
- page 173, Exercise 129. Change the parts **a** and **b** to **b** and **c**.
- page 173, Exercise 134, line 2. Change the period after Petersen to a comma.
- page 176, Exercise 146. Bruce Sagan points out that this exercise can also be done directly from (1.59) and a simple Inclusion-Exclusion argument.
- page 176, line 7-. Change “left subtree” to “right subtree”.

- page 179, line 4. Change $F_1\left(\frac{1}{1+x}\right)$ to $F_1\left(\frac{x}{1+x}\right)$.
- page 179, line 5. Change $G\left(\frac{1}{1+x}\right)$ to $G\left(\frac{x}{1+x}\right)$.
- page 180, Exercise 161(b). Change every appearance of H in this solution to F .
- page 181 line 5. The two square roots should be preceded by \pm since their signs have yet to be determined.
- page 181, Exercise 164, line 1. Change $x = 0$ to $y = 0$.
- page 181, line 4-. Change x to y .
- page 182 Insert after “ $x + p_2x^2 + \dots$.” the following sentence:
Then we need to show that $p_{2n} \in I := \langle p_2, \dots, p_{2n-1} \rangle$, the ideal of the polynomial ring $K[a_2, a_3, \dots]$ generated by p_1, \dots, p_{2n-1} .
The sentence after this one should begin a new paragraph.
- page 183, line 5. Change $A(-A(-x))$ to $A(-A(-x))$.
- page 183, Exercise 168. The parts **b.**, **c.**, **d.**, **e.**, **f.** should be **d.**, **f.**, **g.**, **i.**, **j.**, respectively.
- page 183, line 11-. Change $n \geq$ to $n \geq 0$.
- page 184, line 1. Change xy^2 to x^2y .
- page 186, lines 4–5. Change D to E (twice), since D has already been used.
- page 186, Exercise 164. Change the parts **a** and **b** to **b** and **c**.
- page 186, Exercise 185, line 6. Insert $\prod_{d \geq 1}$ after $=$.
- page 188, Exercise 188, line 4. Change \mathbb{F}_q to \mathbb{F}_q^n (twice).
- page 189, line 18-. Insert the q -binomial coefficient $\binom{n}{r}_q$ after $\sum_{r=1}^n$.
- page 190, Exercise 1.190(b), line 3. Replace this line with

$$\omega^*(n, q) = q^n - q^m - q^{m-1} - q^{m-2} - \dots - q^{\lfloor n/3 \rfloor} + O(q^{\lfloor n/3 \rfloor - 1}).$$

- page 191, third solution to Exercise 191. Change Φ to Φ_M (four times) for notational consistency.
- page 192, lines 3 and 8. Change γ_n to $\gamma(n)$.
- page 192, line 3-. insert “/q” before the period.
- page 193, line 2. Change $\gamma_{n-k}(q)$ to $\gamma(n - k, q)$.
- page 195, line 9-. Change “example” to “examples”.
- page 196, equation (2.5). Change $Y \supseteq T$ to $Y \supseteq \emptyset$ or to just Y .
- page 198, line 3. Change $f = (T)$ to $f_=(T)$.
- page 198, line 2. This line should read:
function $f_ =$ satisfies $f_=(T) = f_=(T')$ whenever $\#T = \#T'$. Thus also $f_{\geq}(T)$
- page 199, line 11-. Change “i set” to “i-set”.
- page 199, line 3-. Change $S - T$ to $S_n - T$.
- page 200, line 8. Change $j \geq i$ to $j \leq i$.
- page 203, line 6. Insert) before }.
- page 203, line 13. Change “nonattacking” to “nonattacking”.
- page 203, line 4-. Change “is” to “in”.
- page 205, line 1. Change “revisted” to “revisited”.
- page 205, line 2-. Change $m - k - 1$ to $m - k + 1$.
- page 206, line 9. Change “suggest” to “suggests”.
- page 207, line 4-. Change $x - s_1$ to $x + s_1$.
- page 208, line 10-. Change “ a_3 's” to “ a_3 3's”.
- page 210, line 9. Insert “one for 121,” after “one for 112,”.
- page 210, line 9-. Insert = between $d(n)$ and $\#D_n$.

- page 212, line 3. Insert a period after $b_1 > a_1$, and raise the end of proof symbol to the same line as the period.
- pages 212–213. There is a simpler involution proof of (2.30) (equivalently, (2.5)). See pages 46–47 of B. Sagan, *The Art of Counting*.
- page 212, line 9–. Change $T \subseteq S$ to $Y \subseteq S$.
- page 216, line 5–. Insert $)$ after x_{γ_i} .
- page 217, l. 14–. Change v to (x, y) .
- page 217, l. 12–. Change v to (x, y) .
- page 222, Exercise 10(b), lines 1 and 2. Change $E(n)/n!$ to $E(n)/n^n$ (twice).
- page 222, Exercise 11. This exercise is stated incorrectly. The correct statement is the following.

- (a) [3–] Fix a subset S of $[n - 1]$. Let a_1, \dots, a_k be the sizes, from left-to-right, of the maximal subsets of S that are sequences of consecutive integers. For instance, if $S = \{2, 5, 6, 7, 9, 12, 13\}$ then $(a_1, a_2, a_3, a_4) = (1, 3, 1, 2)$. Write $\varphi(S) = (a_1, \dots, a_k)$. Let $M(a_1, \dots, a_k)$ denote the number of permutations in \mathfrak{S}_n with descent set S having $n - \#S$ fixed points (the maximum possible). Show that $M(a_1, \dots, a_k)$ is the coefficient of $x_1^{a_1} \cdots x_k^{a_k}$ in the expansion of

$$\frac{1}{(1 + x_1)(1 + x_2) \cdots (1 + x_k)(1 - x_1 - x_2 - \cdots - x_k)}. \quad (1)$$

Example. The coefficient of x_1^{2m} is 1. If $n = 2m + 1$ then this corresponds to the permutation $2m + 1, 2m, \dots, 1$ with one fixed point. The coefficient of x_1^{2m-1} is 0, since there is no permutation in \mathfrak{S}_{2m} with descent set $[2m - 1]$ and one fixed point.

- (b) [1+] Show that $M(a_1, \dots, a_k)$ does not depend on the order of the numbers a_1, \dots, a_k .

(c) [2+] Show that the coefficient of $x_1x_2\cdots x_k$ in the expansion of equation (1) is D_k , the number of derangements in \mathfrak{S}_k . Deduce that the number of alternating permutations in \mathfrak{S}_{2m} (or in \mathfrak{S}_{2m+1}) with m fixed points is D_m .

- page 223, line 4. Change $[n]$ to $[k]$.
- page 223, Exercise 13(b), line 4. Change $S' \supseteq S$ to $S' \subseteq S$.
- page 224, Exercise 14(a), line 8. Insert $+36 \cdot 5^n$ after $-36 \cdot 6^n$.
- page 225, Exercise 18, line 3. To be consistent with Exercise 17, change $w(i+j)$ to a_{i+j} and $w(i)$ to a_i .
- page 227, line 3-. On the left-hand side of the formula, remove one $\sum_{i \geq 0}$ and change y^i to t^i .
- page 229, Exercise 29(a), line 4. Change “partitions” to “compositions”.
- page 235, Exercise 10(b), line 9. Change $E(n)/n!$ to $E(n)/n^n$.
- page 237, Exercise 19. Change $\binom{2n-k}{k}$ to $\frac{2n}{2n-k} \binom{2n-k}{k}$.
- page 237, line 2-. Change second a_{n-k+1} to a_{n-k+2} .
- page 245, line 8. The notation card t is not defined in the text, though it does appear on page 581.
- page 248, line 5-. Change “an” to “a”.
- page 249, Proposition 3.3.2, line 1. Change “condtions” to “conditions”.
- page 251, line 1-. Change \mathcal{I} to S .
- page 252, line 3. Insert “simple” before “matroids”.
- page 253, line 12-. Change t to v (three times).
- page 253, line 9-. In the definition of finitary distributive lattice, add the condition that L has finitely many elements of each rank. (Thus for instance the distributive lattice of all finite subsets of an infinite set is locally finite with $\hat{0}$ but not finitary.)

- page 274, line 1. Change “equation (3.27)” to “the equation above (3.23)”
- page 277, line 12-. Change $\dim(W \cup W')$ to $\dim(W + W')$.
- page 283, lines 8- to 6-. Delete the sentence “Let $\Lambda_t = \{s \in L(\mathcal{A}) : s \leq t\}$, the principal order ideal generated by t .”
- page 284, caption to Figure 3.21. Change \mathcal{A}^K to \mathcal{A}^u .
- page 284, line 5-. In order to make the title “Deletion-Restriction” of Proposition 3.11.5 clearer, one should define \mathcal{A}' to be the *deletion* of H_O from \mathcal{A} and \mathcal{A}'' to be the *restriction* of \mathcal{A} to H_0 .
- page 284, line 3-. Delete “real” at the end of the line.
- page 284, line 2-. Insert “over K ” after “arrangements”.
- page 285, line 9. Delete “ $\#\mathcal{B}_1 = \#\mathcal{B} - 1$ and”.
- page 285, line 11. Under the second Σ , change $\mathcal{B}_1 \in \mathcal{A}''$ to $\mathcal{B}_1 \subseteq \mathcal{A}''$
- page 288, proof of Proposition 3.11.9. Bruce Sagan points out that it would be simpler to say that every subspace in $L(\mathcal{A})$ over \mathbb{Z} is defined by a row-reduced echelon matrix over \mathbb{Z} , so we have good reduction over any prime p larger than all the entries of this matrix.
- page 291, line 10-. Change “sketch that” to “reference for”.
- page 303, Theorem 3.15.8. To be completely accurate, one should assume that $P \neq \emptyset$.
- page 308, line 11. Insert “and equation (3.67)” after 3.15.7.
- page 308, line 9. Change “an” to “and”.
- page 311, line 11 (beginning $n = 6$): change 0 to 2.
- page 312, line 12-. Change the second $=$ to $-$.
- page 313, Figure 3.33. The poset P_5 is drawn incorrectly.
- page 315, line 7. Change $a_s - a_{s+1}$ to $a_s - a_{s-1}$ (twice after the brace).

- page 317, after (3.80). The phrase “(since intervals of Eulerian posets are Eulerian)” is unnecessary since the formulas under consideration hold for any graded poset with $\hat{0}$ and $\hat{1}$.
- page 319, line 12–. Change $Q_1 * Q_2 * \cdots * Q_r$ to $R_1 * R_2 * \cdots * R_r$ (since Q_i already has been given another meaning).
- page 319, line 11–. Change the first Q_i to R_i .
- page 332, line 1–. Change γ_{j-1} to γ_{j-1}^2 .
- page 332, bottom of page. Add the following paragraph:
The proof that $(\gamma_j^*)^2 = 1$ is completely analogous. It also follows from the fact that $\gamma_j^2 = 1$, since we can assume without loss of generality that $j = p - 1$ and then apply the automorphism of G that sends each τ_k to τ_{p-k} .
- page 334, line 4. Change $z\delta$ to $z\delta_{p-1}$.
- page 336, line 11. Change $\varphi: \widehat{KP} \rightarrow \widehat{KP}$ to $\varphi: KP \rightarrow \widehat{KP}$. page 337, line 3–. Change $f(U)$ to $f(y)$.
- page 337, lines 14– to 11–. The sentence “For the algebraically minded . . . formal power series.” is not correct. We first need to consider non-commutative *polynomials* (not power series) and then pass to suitable completions.
- page 338, line 1–. This statement might not be so clear. Now $e^{Dx}f(U)$ is an operator, operating on power series in U , where D acts as $r\frac{d}{dU}$. Let $e^{Dx}f(U)$ act on $g(U)$. The Taylor series expansion gives

$$\begin{aligned} e^{Dx}f(U)g(U) &= f(U + rx)g(U + rx) \\ &= f(U + rx)e^{Dx}g(U), \end{aligned}$$

so $e^{Dx}f(U) = f(U + rx)e^{Dx}$.

- page 342, line 6. Change $(1 - q)^2$ to $(1 - q)$ (in the denominator).
- page 345, line 4. Change $p_{s-1} + p_s$ to $p_{j-1} + p_j$.
- page 345, line 13. Change A to **A** (boldface).

- page 353, Exercise 3(d).
- page 354, Exercise 7(a), line 1. Change “the longest” to “a longest”.
- page 356, Exercise 15(g), line 3. Change 9655 to 11586.
- page 356, Exercise 16(b). Change the difficulty rating to [3] (or maybe [3+]). See M. Guay-Paquet, A. H. Morales, and E. Rowland, DMTCS proc. **AS** (FPSAC 2013 Paris) (2013), 253–264.
- page 358, Exercise 31(b,c,d). It should be assumed that L has rank at least three.
- page 360, Exercise 38. This exercise is incorrect for $1 < k < m$, where m is the maximum size of an antichain of P . A counterexample is given by $k = 2$ and $L = J(P)$, where $P = \mathbf{1} + (\mathbf{1} \oplus (\mathbf{1} + \mathbf{1}))$. It is true that $\#P_k = \#R_k$, even for modular lattices. See Exercise 3.101(d).
- page 360, Exercise 41(c,d). It is assumed that L is distributive.
- page 360, Exercise 41(d), line 2. after t_i insert “, $0 \leq i \leq m$ ”.
- page 363, Exercise 49, line 3. Change 3.6 to 3.44. (Figure 3.6 is in fact the dual of Figure 3.44.)
- page 363, Exercise 50, line 4. Change $i \leq j \leq 0$ to $i \geq j \geq 0$.
- page 363, Exercise 51, line 2. Not an error, but to be consistent with the rest of the exercise it would be better to use n rather than i (twice).
- page 364, Exercise 55(b). The rating should be changed to [3–]. An exceptionally elegant proof was given by G. Stachowiak, *Order* **5** (1988), 257–259. Another elegant proof was given by B. Iriarte, [arXiv:1405.4880](https://arxiv.org/abs/1405.4880). Iriarte proves the more general result that if G is the comparability graph of a poset P , then the number of linear extensions of (G, \mathfrak{o}) is maximized when \mathfrak{o} respects the order P .
- page 366, Exercise 62(g), line 2. Change $m \geq 0$ to $n \geq 0$ (under the summation sign).

- page 366, Exercise 62(g), line 2. Change $U_{P_n}(x)$ to $G_{P_n}(x)$, where $G_{P,\omega}(x)$ is defined in Section 3.15.2, and as usual we suppress the notation ω in the case of natural labelings. Equivalently, for any finite labelled poset (P, ω) ,

$$U_{P,\omega}(x) = \lim_{m \rightarrow \infty} U_{P,\omega,m}(x).$$

- page 374, Exercise 95. The stated identity is actually valid for the *dual* Möbius algebra, where one uses the join operation rather than the meet. The correct identity using the text definition of Möbius algebra is

$$\sum_{t \in L} \mu(t, \hat{1})t = \left(\sum_{u \geq z} \mu(u, \hat{1})u \right) \cdot \left(\sum_{v \vee z = \hat{1}} \mu(v, \hat{1})v \right).$$

- page 375, Exercise 96(d). This result was proved by Braden, Huh, Matherne, Proudfoot and Wang (difficulty rating [4]). For a survey, see T. Braden, J.P. Matherne, and N. Proudfoot, *Notices AMS*, to appear; <https://pages.uoregon.edu/njp/whatisNotices.pdf>.
- page 375, Exercise 98, line 3. Change \leq to \geq .
- page 376, Exercise 101(f). Difficulty level [2+].
- page 378, Exercise 114(a). The answer should be

$$(x-1)(x-2) \cdots (x-n).$$

- page 379, Exercise 115(c), line 1. Change “0,1” to 0. (There are $\binom{n}{2}$ hyperplanes.)
- page 379, Exercise 116. The subscripts should not be in boldface.
- page 385, Exercise 131, lines 10–13. The definition of the partial ordering is incorrect. We should define $\alpha \leq \beta$ if every block $B \in \beta$ is a union of blocks A_{i_1}, \dots, A_{i_k} of α (and that the restrictions of the labelings agree).
- page 387, Exercise 3.139. The right-hand side of the displayed formula should be $2x(1+x)^{n-2}/(1-x)^n$.

- page 387, Exercise 141, line 3. Change $\sum_{i \geq 1}$ to $\sum_{i \geq 0}$.
- page 387, line 7-. It should be said that Q_0 is defined in the previous exercise.
- page 388, Exercise 145, line 3. Change 3.15.10 to 3.15.12.
- page 388, Exercise 149, line 4. Change $1 \leq i, k \leq r$ to $1 \leq i, j \leq r$.
- page 389, Exercise 155, lines 2 and 5. The hypothesis that every interval is self-dual can be weakened to every interval being rank-symmetric.
- page 390, Exercise 158(a), line 4. Change **a.** at beginning of line to **i.**
- page 390, Exercise 158(a), line 5. Change **b.** at beginning of line to **ii.**
- page 390, Exercise 158(c). Delete one of the periods at the end of the line.
- page 390, Exercise 158(d), line 2. Change “thats” to “that”.
- page 390, Exercise 158(e), line 1. Change $k(kt)!^{t-2}$ to $k(kt)^{t-2}$.
- page 393, Exercise, 166, line 3. Change the first x to \mathbf{x} .
- page 394, line 7-. Change $q^{\binom{b+1}{2}}$ to $q^{-\binom{b+1}{2}}$
- page 401, line 2-. Change \mathfrak{S}_n to $W(\mathfrak{S}_n)$.
- page 402, Exercise 187, line 4. Change $\mathbf{2} \times \mathbf{n}$ to $\mathbf{2} \times (\mathbf{n} - \mathbf{1})$.
- page 403, Exercise 189. NOTE. The result is also true for d odd, but the proof is quite a bit easier.
- page 403, Exercise 190(b). The definitions of P_2 and P_3 are not clear. The generating function $1/(1-2x-2x^2)$ gives the right values for $M(n)$ when $n \geq 3$.
- page 404, Exercise 198. It should be mentioned that butterfly posets are also called “ladder posets,” since there is an index entry for “poset, ladder.” This terminology should have been introduced in Exercise 3.176(a), since that is where they first appear.

- page 405, Exercise 201(b), line 1. Change $B(n, n + 1) - 1$ to $B(n, n + 2) - 1$.
- page 405, line 8-. The notation f_{00} is not defined until the next sentence.
- page 406, line 1-. Change $\text{des}(w)$ to $\text{asc}(w)$.
- page 406, line 7-. The condition $s < t$ is not necessary; it is implied by $\ell(s, t) = n \geq 1$.
- page 407, line 1. Change $\text{des}(w)$ to $\text{asc}(w)$, and change “descents” to “ascents”.
- page 407, line 3. Change $z \sum_{n \geq 1}$ to $\sum_{n \geq 1}$ (twice).
- page 407, line 5. Change $(x - 1)^{n-1}$ to $(z - 1)^{n-1}$.
- page 407, Exercise 205(b), line 2. Change 44605 to 44606 (private communication from Patrick Byrnes, 21 February 2012). Byrnes originally assumed that a vertex v could be covered by at most one singleton (element covering only v), but there is exactly one example up to rank 9 where this property need not hold. Byrnes also computes that there are 29,199,636 1-differential posets up to rank 10.
- page 408, Exercise 211, line 3. Change the numerator $r^2 + (r + 1)q - q^2$ to $r^2 + r(r + 1)q - rq^2$.
- page 408, Exercise 215(c). Change the difficulty rating [5] to [3]. See A. Miller, *Order* **30** (2013), 657–662, arxiv.org/abs/1202.3006, and C. Gaetz and P. Venkataramana, *Order* **37** (2020), 279–286; arxiv.org/abs/1806.03509.
- page 411, line 8-. Change **f.** to **g.**.
- page 411, line 8-. Change (e) to (f).
- page 412, Exercise 16, line 2. Update the reference to *J. Combin. Theory Ser. A* **120** (2013), 1305–1327.
- page 413, Exercise 26, line 2. We can’t replace \mathbb{N}^* with \mathbb{Z} since \mathbb{Z} doesn’t have a $\hat{1}$.

- page 415, Exercise 36(b). This example is incorrect. For instance, $\{1, 2, 3\}$ and $\{1, 2, 4, 5\}$ have no meet.
- page 416, Exercise 41(a), line 3. Change “dominance” to “lexicographic”.
- page 416, Exercise 47(b). Bruce Sagan points out that this exercise can be given a more intuitive, noninductive proof.
- page 418, line 5. To verify this identity, we must use the fact that $2 \leq r \leq k$. (We are assuming $r > 1$, and if $r > k$ then the meet of elements covered by t has rank $k - r < 0$.)
- page 426, Exercise 87(d), line 1. In addition to Example 3.9.6, we are also using the fact (immediate from the definition of meet-distributive) that if $[s, t]$ is an interval of a meet-distributive lattice for which s is the meet of elements in $[s, t]$ covered by t , then $[s, t]$ is boolean, together with Corollary 3.9.5.
- page 426, Exercise 87(d). One should define a chain $t_0 < t_1 < \cdots < t_k$ to be *boolean* if every interval $[t_{i-1}, t_i]$ is a boolean algebra. Also define ℓ to be the length of the interval $[t_0, t_k]$.
- page 426, Exercise 88. Another solution is the following. We have

$$\sum_{s \leq t} \mu(s, t) = \sum_s \sum_{t \geq s} \mu(s, t).$$

By the definition (3.15) of μ , this latter sum over t is 0 if $t \neq \hat{1}$ and is 1 if $t = \hat{1}$, and the proof follows. Note that this proof only requires P to have a $\hat{1}$. More generally, we leave the reader to show that for *any* finite poset P , $\sum_{s \leq t} \mu(s, t) = 1 + \mu_{\hat{P}}(\hat{0}, \hat{1})$, where \hat{P} denotes P with a $\hat{0}$ and $\hat{1}$ adjoined.

- page 427, Exercise 96(a), line 7. Change $[F(s \wedge t), s]$ to $[F(s \wedge t, s)]$.
- page 429, line 9. Change “that” to “then”.
- page 429, line 12. The sum is not empty, but all terms u satisfy $u \notin B$. Hence $f(u) = 0$.

- page 429, line 13–. Change T^* to t^* .
- page 430, Exercise 102(a). Considerable further progress (but not a complete solution) has been made. See the Wikipedia article “Union-closed sets conjecture.”
- page 431, line 5. Change $v \geq u$ to $v \leq u$.
- page 431, line 2–. Change **23** to **33**.
- page 434, line 9–. Change $p > 2$ to $p > 2n - 1$.
- page 437, line 11. Not the best notation, since $x||y$ also means that x and y are incomparable in a poset.
- page 439, Exercise 130, line 1. insert $-\{\hat{0}\}$ after Q_i .
- page 440, Exercise 135, line 1. Change 3.12 to 3.13.
- page 445, Exercise 151, line 3. Change the second $+1$ (subscript) to $+2$.
- page 449, Exercise 163(a), line 3. Change $2e_P(p-1) - (p-1)e(p)$ to $(2e_P(p-1) - (p-1)e(P))/2(p-1)!$.
- page 450, lines 8– and 4–. Change C_k to C_m .
- page 451, Exercise 170(c), line 2. Change n to $n + 1$.
- page 451, Exercise 170(c), line 3. Change q^n to $q^{n+1}ecur$.
- page 452, lines 5– and 3–. Change h_i to h_j .
- page 453, line 6. Change $U_{m,P}(q)$ to $U_{P^*,m}(q)$.
- page 453, line 10–. Change $1 - q^{p+1}y$ to $1 - q^{p+1}$.
- page 453, line 8–. One also needs the unlabeled equation between (3.148) and (3.149).
- page 453, line 3–, and page 454, line 1. Change W_i to $W_{P,i}(q)$ and W_{i-1} to $W_{P,i-1}(q)$.
- page 454, line 3. Change W_i to $W_{P,i}(q)$.

- page 455, Exercise 175. Bruce Sagan points out that a short combinatorial proof can be given using Exercise 174(a).
- page 458, Exercise 185. Part **f.** should be **k.**, and **g.** should be **l.**
- page 459, Exercise 193(a), lines 1–2. It would make more sense to introduce the terminology “butterfly poset” in Exercise 3.176 and to replace 3.198 with 3.176.
- page 459, line 5–. Should be $\beta_P(4, 5, 6) = -1$.
- page 459, line 3–. Exercise 3.155(a) assumes that every interval is self-dual, which is stronger than rank-symmetric. However, Exercise 3.155(a) is valid if one assumes only rank-symmetry. See the entry for page 389, Exercise 155, in this Errata.
- page 460, Exercise 196(b), lines 2– to 1–. A more natural combinatorial interpretation and generalization of $\binom{n}{k}_F$ is given by C. Bennett, J. Carrillo, J. Machacek, and B. E. Sagan, *Ann. Comb.* **24** (2020), 503–530; arxiv.org/pdf/1809.09036.
- page 462, Exercise 207, line 4. Change 3.21.3 to 3.21.4.
- page 463, line 3. Insert (w) after $c_{i+1,0}$.
- page 463, line 4. Insert “, where $n = \rho(t)$ ” after $U^n(\hat{0})$.
- page 463, Exercise 2.15(a). We could simply appeal to Theorem 3.21.12 since eigenvalue multiplicities are nonnegative. However, the injectivity of U_i is part of the proof of Theorem 3.12,12, so for a proof from scratch of the present exercise it is simpler to use the injectivity of U_i .
- page 465, line 7–. Change “degree j ” to “degree $j - 1$ ”.
- page 468, Proposition 4.2.2, line 5. Change the period after E_f to a comma.
- page 469, line 6–. Change $\sum_{n \geq 0} f(n)$ to $\sum_{n \geq 0} f(n)x^n$.
- page 470, line 5–. Change β_s to β_{d-s} .
- page 470, line 1–. Change β^r to β_r .

- page 471, line 10–. Insert “for n sufficiently large” after ζ_i^n .
- page 479, (4.17). Note that we could replace Γ by Γ° (both are correct).
- page 484, line 8. Change x^β to \mathbf{x}^β .
- page 487, Lemma 4.5.17(ii). Insert “, where the *degree* of a rational function $G(\lambda) = P(\lambda)/Q(\lambda)$ is defined by $\deg G = \deg P - \deg Q$ ” after 0.
- page 487, line 3–. Change D_S to D_F .
- page 489, line 16. Change the first x_{ij} to x_{1j} .
- page 489, line 11–. Change 4.6.16 to 4.5.16.
- page 489, line 3–. Change $n - 1$ to $-n + 1$.
- page 490, line 5. Insert “for r sufficiently large” after “choose”.
- page 492, line 7–. Change the first x_{ij} to x_{1j} .
- page 493, line 9–. Change $\text{conv } V$ to $\text{conv } S$.
- page 494, line 5–. Change \mathbb{R}^m to $\mathbb{R}_{\geq 0}^m$.
- page 497, line 2. Change $i(\mathcal{P}, n)$ to $i(\mathcal{P}, -n)$.
- page 497, line 1–. Change $n \in \mathcal{P}$ to $n \in \mathbb{P}$.
- page 498, line 1. Change $m\gamma$ to $n\gamma$.
- page 498, line 1–. Change $A_d(1)$ to $A(\mathcal{P}, 1)$.
- page 503, Example 4.7.5. Though the point of this example is to illustrate the transfer-matrix method, it should be pointed out (as suggested by Bruce Sagan) that it is easy to prove combinatorially that $f_p(n) = p(p-1)^{n-1} - f_p(n-1)$ and then solve this recurrence.
- page 504, line 3–. Change $1 - \lambda A$ to $I - \lambda A$.
- page 505, line 6–7. It would be better to use the initial conditions $f(0) = 1, f(1) = 3, f(2) = 7$.

- page 506, Figure 5.8. Replace the edges between 32 and 22, and between 23 and 33, with arcs from 32 to 22 and from 23 to 33.
- page 511, line 10. Replace $\#A$ by its weighted analogue, namely, $\sum_{(u,i) \in A} w(u)$, and similarly for $\#B$.
- page 511, line 18-. Change (v, b_i) to (v, i) , $1 \leq i \leq k$.
- page 511, line 14-. Change (v, b_i) to (v, i) .
- page 511, Example 4.7.15. It should be stated that we are taking each $w(u) = 1$.
- page 513, line 2-. Change “amd” to “and”.
- page 522, Figure 4.29. The vertex labels should be 0 and 1, not 00 and 10.
- page 528, Exercise 4.2, line 3. Change “over” to “in”.
- Exercises 7(a), line 2, and 7(b), line 2. Change Σ to Π .
- page 530, Exercise 4.12. Change $0.00010203050813213455 \dots$ to $0.0001010203050813213455 \dots$, and change 1, 2, 3 to 1, 1, 2, 3.
- page 530, Exercise 4.14, line 2. Insert “relatively prime” after “choose a, b ”.
- page 531, Exercise 22, line 1. Change $\alpha \in \mathbb{C}$ to $0 \neq \alpha \in \mathbb{C}$.
- page 532, Exercise 25(d). This should have a difficulty rating of [3-].
- page 532, Exercise 25(e), line 3. Change $N_1(m)$ to $N_1(2^m - 1)$.
- page 532, Exercise 25(e), line 5. Change $N_1(m)$ to $N_1(q^m - 1)$ and $N_{-1}(m)$ to $N_{-1}(q^m - 1)$.
- page 532, Exercise 25(e), line 8. Change $N_1(m)$ to $N(2^m - 1)$.
- page 532, Exercise 25(e), line 10. Change $N_1(m)$ to $N_1(2^m - 1)$.
- page 533, line 2. Change $N_1(m)$ to $N_1(2^m - 1)$.
- page 533, line 5. Change $N_1(m)$ to $N_1(2^m - 1)$.

- page 533, line 7. Change $N_1(m)$ to $N_1(3^m - 1)$.
- page 533, line 8. Change $N_2(m)$ to $N_2(3^m - 1)$.
- page 533, line 10. Change $N_1(m)$ to $N_1(3^m - 1)$.
- page 533, line 11. Change $N_2(m)$ to $N_2(3^m - 1)$.
- page 534, Exercise 28(b). This should have a difficulty rating of [2].
- page 534, line 5-. Change “the number” to “the minimal number”.
- page 535, Exercise 37. Throughout this exercise, x , α_i and γ should be in boldface.
- page 537, line 6-. Insert “of length m ” after $w \in X^*$.
- page 539, Exercise 47, line 1-. Change 1.1.8.6 to 1.8.6.
- page 540, Exercise 53(b), line 4. Change n to d (twice).
- page 542, line 4. Change $n \times n$ to $(n + 1) \times (n + 1)$.
- page 542, line 10. Delete) after u_6 .
- page 544, Exercise 4.66(b), line 2. Change “each i -face” to “the affine span of each i -face”.
- page 546, Exercise 4.75(d). Change the difficulty rating to [3-]. A combinatorial proof (though not a simple bijection) was found by Jacob Scott, private communication dated September 2, 2012.
- page 548, Exercise 2(a), line 6. The assertion “Clearly, we can write $f(x) = P(x)/Q(x)$ for some relatively prime integer polynomials P and Q .” is not so clear. One must show that if $F(x) \in \mathbb{Q}[[x]]$ and $F(x) = R(x)/S(x)$ where $R, S \in \mathbb{C}[x]$, then one can write $F(x) = P(x)/Q(x)$ where $P, Q \in \mathbb{Q}[x]$. This statement does have a fairly simple proof, which we leave as an exercise.
- page 549, Exercise 3, line 6. Change “polynomials” to “formal power series”.
- page 549, line 3-. Change $\frac{1}{(1-x)^2(1-y)}$ to $\frac{x}{(1-x)^2(1-y)}$.

- page 549, line 1–. Change $\sum x^{m^2} y^n$ to $F(x, y) - \sum x^{n^2} y^n$.
- page 550, Exercises 7(a) and 7(b). Bruce Sagan points out that rather than invoking Example 1.1.14, it is easier simply to use the product formula for a derivative.
- page 550, Exercise 5.50, line 1. It would be more accurate to change the first sentence to: Given $\alpha \in \mathbb{P}$, $\alpha \geq 2$, define a_i so that

$$F(x) := \prod_{i \geq 1} (1 - x^i)^{-a_i} = \frac{1}{1 - \alpha x}.$$

(Unique such a_i exist by Exercise 1.158.)

- page 551, Exercise 9, line 1–. Change “integer” to “integers”.
- page 552, Exercise 14. The smallest known pair (a, b) seems to be

$$\begin{aligned} a &= 106276436867 = 31 \cdot 3128272157 \\ b &= 35256392432 = 2^4 \cdot 2203524527 \end{aligned}$$

due to M. Vsemirnov, *J. Integer Seq.* **7** (2004), article 04.3.7.

- page 554, line 1–. Change xy to $2xy$.
- page 555, line 2. Change Theorem 4.1.1(iii) to Proposition 4.2.2(iii).
- page 557, line 4–. Change x^ℓ to x^l .
- page 558, line 1. Delete comma after *Wochenschrift*.
- page 558, Exercise 43. The sequence $t(3), t(4), \dots$ is known as *Alcuin’s sequence*, after Alcuin of York (730s or 740s – 19 May 804). For a survey see D. J. Bindner and M. Erickson, *Amer. Math. Monthly* **119** (2012), 115–121. In addition, Bruce Sagan has given a simple bijection between the partitions of n into parts 2, 3, and 4 with at least one part equal to 3, and partitions counted by $t(n)$.
- page 560, Exercise 46(b), line 1. Delete the first “many”.
- page 560, Exercise 46(b), line 2. Change $f(n+1), f(n+2), \dots, f(n+d)$ to “ $(f(n+1), f(n+2), \dots, f(n+d))$ for fixed d ”.

- page 561, line 3. Change $\sum_{j=0}^{d+1}$ to $\sum_{j=0}^d$.
- page 561, line 12. Although this formula is correct, it makes more sense to change $\sum_{k=0}^{r+s}$ to $\sum_{k=0}^r$.
- page 561, line 12. Change $V(\mathcal{P})$ to $\nu(\mathcal{P})$.
- page 561, Exercise 51(a). The volume is easy to see directly, since \mathcal{P} is a cartesian product of two simplices with volumes $1/r!$ and $1/s!$.
- page 561, Exercise 53(c). Change each $\binom{*}{*}$ to $\binom{*}{*}$ (four times).
- page 562, line 7. Equation (4.29) is stated only for $r \in \mathbb{Z}$, but it holds for $r \in \mathbb{C}$ since a polynomial in $\mathbb{C}[r]$ has only finitely many zeros. It then follows that the average of the zeros of $H_n(r)$ is $-n/2$.
- page 562, Exercise 56(b), line 3. We also need to define $\delta_i = \varepsilon_i$ when t_i is a minimal element of P .
- page 564, Exercise 58(f), line 1. Change “order polynomials” to “Ehrhart polynomials”.
- page 564, line 7. Insert “nonempty” after “every”.
- page 564, line 15. Insert “nonempty” after “each”.
- page 567, Exercise 75(c), line 1. Delete “directed”.
- page 569, Exercise 80, displayed equation, line 1. Change $v'_i y$ to $v_i y$.
- page 570, Exercise 82(e,f). For the paper of J. Schneider, see *Electronic J. Combinatorics* **21** (2014), P1.43; [arXiv:1206.6174](https://arxiv.org/abs/1206.6174).
- page 571, line 3. Change “chose” to “chosen”.
- page 578. The following item is missing: First edition—Supplementary Exercise 1.7 = Second edition—Exercise 1.114(b).
- page 579. The following three items are missing: First edition—Supplementary Exercise 3.19 = Second edition—Exercise 3.44; First edition—Supplementary Exercise 3.10 = Second edition—Exercise 3.63; and First edition—Supplementary Exercise 3.17 = Second edition—Exercise 3.189.

- page 620, Stembridge entry, line 6. Change $q = 1$ to $q = -1$.